CENTER FOR
RISK MANAGEMENT
RESEARCH

Working Paper #2017-***02***

Equilibrium Comparative Statics in Finite Horizon Finance Economies with Stochastic Taxation

Konstantin Magin, University of California, Berkeley

April 21, 2017

University of California
Berkeley

Equilibrium Comparative Statics in Finite Horizon Finance Economies with Stochastic Taxation*

Revised from the Center for Risk Management Research Working Paper 2016-03

Konstantin Magin†

April 21, 2017

Abstract

This paper studies equilibrium comparative statics of Financial Markets (FM) equilibria in the finite horizon General Equilibrium with Incomplete Markets (GEI) model with respect to changes in stochastic tax rates imposed on agents' endowments and dividends. We show that under reasonable assumptions, without assuming CRRA and identical agents, an increase in the current dividend tax rate unambiguously reduces current asset prices. The paper also finds that there exists a bound $\overline{B}$ such that for a coefficient of relative risk aversion less than $\overline{B}$, an increase in a future dividend tax rate reduces current price of tradable assets. At the same time, for a coefficient of relative risk aversion greater than $\overline{B}$, an increase in a future dividend tax rate boosts the current price of tradable assets. Finally, for a coefficient of relative risk aversion equal to $\overline{B}$, an increase in a future dividend tax rate leaves current price of tradable assets unchanged. As a special case, under additional assumptions, $\overline{B}$ is equal to 1. Also, under reasonable assumptions, an increase in the current endowment tax rate reduces current asset prices, while an increase in a future endowment tax rate boosts current asset prices.

Keywords: Comparative Statics, Stochastic Taxation, GEI, Complete Markets, CCAPM, Property Rights, Equity Premium, Risk Aversion

JEL Classification: D5; D9; E13; G12; H20.

*This research was supported by a grant from the Center for Risk Management Research. I am very grateful to Bob Anderson for his kind support, encouragement and insightful conversations.

†The University of California at Berkeley, The Center for Risk Management Research, 579 Evans Hall, #3880, Berkeley, California 94720-3880, e-mail: magin@berkeley.edu

1. INTRODUCTION

Taxes are a part of individuals' and corporations' budget constraints. Therefore, taxes clearly affect equilibrium commodity and asset prices and allocations. Also, changes in various tax rates, especially income tax, are driven by the constantly changing political balance of power, and the direction of those changes seems to have been anything but predictable. Thus, it seems entirely appropriate to regard future taxation as stochastic.

But if taxation is stochastic, then it is clearly a risk factor affecting equilibrium asset prices through stochastic discount factors and after-tax dividends. Since this risk cannot be eliminated or substantially reduced by diversification, standard finance theory suggests that it ought to be an asset-pricing risk factor, which ought to affect asset prices and allocations.

Surprisingly, however, there has been very little research done to date on the effects of stochastic taxes on equilibrium asset prices and allocations. The research done so far, relies on the CCAPM with identical agents and twice-differentiable utility functions and focuses primarily on resolving the so-called "Equity Premium Puzzle." See Magin (2015a), Edelstein and Magin (2016) and (2013), DeLong and Magin (2009), Sialm (2009) and (2006). While resolving the Equity Premium Puzzle is critically important for confirming the validity of the Lucas-Rubenstein CCAPM with identical agents, the role of insecure property rights (stochastic taxation) in economic theory is much broader. For example, do Financial Markets (FM) equilibria exist in the finite horizon General Equilibrium of Incomplete Markets (GEI) model with stochastic taxation? Do sufficiently small changes in stochastic tax rates preserve the existence and completeness of FM equilibria? Magin (2015b) finds that under reasonable assumptions, FM equilibria exist for all stochastic tax rates imposed on agents' endowments and dividends except for a closed set of measure zero. Moreover, sufficiently small changes in stochastic taxation preserve the existence and completeness of FM equilibria. The next natural question to ask would be: Does an increase in current and future taxes reduce current prices of tradable assets?

This paper studies comparative statics of FM equilibria in the finite horizon GEI model with respect to changes in stochastic tax rates imposed on agents' endowments and dividends. The Sonnenschein-Mantel-Debreu Theorem states that if we exclude prices close to zero, then no further restrictions other than Continuity, Homogeneity and Walras' Law can be imposed on the aggregate excess demand function of an exchange economy. As a result, comparative statics results are fairly rare in general equilibrium (GE).[1] Given the strong methodological connection between GE and GEI, it is not surprising that comparative statics results are also rare in GEI. Here, we develop a technique which we believe to be new, and which is potentially applicable in other situations. We show that, although the sign of a derivative of a complex object of interest may be ambiguous, as is typically the case in GEI, the sign of the derivative of this complex object of interest is always the same as that of the derivative of some other simple and more intuitive object. While the signs of the derivative of this simple and more intuitive object in GEI are often indeterminate, many of them have a natural sign; the presence of the opposite sign is viewed as possible but somewhat rare and pathological. Our methods give intuitive signs to derivatives in cases where the natural sign may not be obvious, by showing that they are the same as the signs in cases where there is an obvious natural sign.

The first major finding of this paper is Theorem 2.2.3. It analyzes comparative statics of current asset prices with respect to changes in current dividend taxes. Dividends are paid in units of the numeraire good 1. It states that although the sign of a derivative of current equilibrium asset prices with respect to current dividend tax rates may be ambiguous, it is always the same as that of the derivative of the current equilibrium consumption of the numeraire good with respect to current dividend tax rates. Specifically, if the numeraire good is a normal good, then under reasonable assumptions, without assuming CRRA and identical agents, an increase in the current dividend tax rate reduces current asset prices. Otherwise, if the numeraire good is an inferior good, then an increase in the current dividend tax rate boosts current asset

[1] See sections 17.E-17.G in Mas-Colell, Whinston and Green (1995) for discussion. Quah (2003) is a rare exception.

prices. While inferior goods have been demonstrated to exist, it is generally believed that they are rare: at any given time and price level, the demand for the vast majority of goods moves in the intuitive direction with respect to changes in after-tax income.[2] So it is reasonable to assume that the numeraire good is a normal good. Therefore, it is natural to conclude that under reasonable assumptions, an increase in the current dividend tax rate reduces current asset prices.

Corollary 2.2.5. of the above theorem states that although the sign of a derivative of before-tax and after-tax rates of return for tradable assets with respect to current dividend tax rates may be ambiguous, it is always the opposite of that of the derivative of the current equilibrium consumption of the numeraire good with respect to current dividend tax rates. Specifically, if the numeraire good is a normal good, then under reasonable assumptions, without assuming CRRA and identical agents, an increase in the current dividend tax rate boosts before-tax and after-tax rates of return for tradable assets. Otherwise, if the numeraire good is an inferior good, then an increase in the current dividend tax rate reduces before-tax and after-tax rates of return for tradable assets. Since it is reasonable to assume that the numeraire good is a normal good, it is natural to conclude that under reasonable assumptions, an increase in the current dividend tax rate boosts before-tax and after-tax rates of return for tradable assets. Assuming that the real risk-free rate of return is constant, we can also conclude that an increase in the current dividend tax rate boosts before-tax and after-tax risk premiums for tradable assets.

The second major finding of this paper is Theorem 2.2.6. It analyzes comparative statics of current asset prices with respect to changes in future dividend taxes. It states that there exists a bound $\overline{B}$ such that for a coefficient of relative risk aversion less than $\overline{B}$, an increase in a future dividend tax rate reduces current prices of tradable assets. At the same time, surprisingly, for a coefficient of relative risk aversion greater than $\overline{B}$, an increase in a future dividend tax rate boosts current prices of tradable assets. Finally, for a coefficient of relative risk aversion equal to $\overline{B}$, an increase in a future dividend tax rate leaves current prices of tradable assets unchanged. Corollary 2.2.8. states that, as a special case, under additional assumptions, $\overline{B}$ is equal to 1.

Theorem 2.3.2. analyzes comparative statics of current asset prices with respect to changes in current endowment taxes. It states that although the sign of a derivative of current equilibrium asset prices with respect to current endowment tax rates may be ambiguous, it is always the same as that of the derivative of the current equilibrium consumption of the numeraire good with respect to current endowment tax rates. Specifically, if the numeraire good is a normal good, then under reasonable assumptions, without assuming CRRA and identical agents, an increase in the current endowment tax rate reduces current asset prices. Otherwise, if the numeraire good is an inferior good, then an increase in the current endowment tax rate boosts current asset prices. Since it is reasonable to assume that the numeraire good is a normal good, it is natural to conclude that under reasonable assumptions, an increase in the current endowment tax rate reduces current asset prices.

Theorem 2.3.4. analyzes comparative statics of current asset prices with respect to changes in future endowment taxes. It states that although the sign of a derivative of current equilibrium asset prices with respect to future endowment tax rates may be ambiguous, it is always the opposite to that of the derivative of the future equilibrium consumption of the numeraire good with respect to future endowment tax rates. Specifically, if the numeraire good is a normal good, then under reasonable assumptions, without assuming identical agents, an increase in the future endowment tax rate boosts current asset prices. Otherwise, if the numeraire good is an inferior good, then an increase in the future endowment tax rate reduces current asset prices. Since it is reasonable to assume that the numeraire good is a normal good, it is natural to conclude that under reasonable assumptions, an increase in the future endowment tax rate boosts current asset prices. Theorem 2.3.5. obtains a similar result by assuming identical agents but without necessarily assuming CRRA.

The paper is organized as follows. Section 2 studies comparative statics of FM equilibria with respect

[2] In his 1957 book "A Theory of the Consumption Function" Milton Friedman develops the Permanent Income Hypothesis and establishes a strong positive correlation between the permanent consumption and the permanent income.

to changes in stochastic taxation of endowments and dividends. Section 3 concludes.

2. COMPARATIVE STATICS OF FM EQUILIBRIA WITH STOCHASTIC TAXATION OF DIVIDENDS AND ENDOWMENTS

2.1. Definitions

First, we need to introduce several definitions to incorporate stochastic taxation imposed on agents' endowments and assets' dividends and used to finance public good G into the General Equilibrium Theory of Financial Markets.[3] Let ET be the event-tree, I be the set of finitely living investors-consumers, L be the set of commodities traded on spot markets, K be the set of assets traded on financial markets, such that

$$max\left[|ET|,\ |I|,\ |L|,\ |K|\right] < \infty.$$

Let $\tau_{e_i} = \{\tau_{e_i}(\xi,\ l)\}_{(\xi,\ l)\in ET\times L} \in [0,\ 1]^{|ET\times L|}$ be the stochastic tax imposed on the individual endowment of agent $i \in I$,

$\tau_{e_i}(\xi) = \{\tau_{e_i}(\xi,\ l)\}_{l\in L} \in [0,\ 1]^{|L|}$ be the vector of the stochastic tax imposed on the individual endowment of agent $i \in I$ at node $\xi \in ET$,

$e_i(\tau_{e_i}) = \{e_i(\xi,\ l,\ \tau_{e_i})\}_{(\xi,\ l)\in ET\times L} \in \mathbb{R}_+^{|ET\times L|}$ be the individual endowment of agent $i \in I$,[4]

$e_i(\xi,\ \tau_{e_i}) = \{e_i(\xi,\ l,\ \tau_{e_i})\}_{l\in L} \in \mathbb{R}_+^{|L|}$ be the vector of the individual endowment of agent $i \in I$ at node $\xi \in ET$,

$\tau_e = \{\tau_{e_i}\}_{i\in I} \in [0,\ 1]^{|ET\times L\times I|}$ be the matrix of taxes imposed on individual endowments,

$e\left(\tau_e\right) = \{e_i(\tau_{e_i})\}_{i\in I} \in \mathbb{R}_+^{|ET\times L\times I|}$ be the matrix of before-tax individual endowments,

$c_i = \{c_i(\xi,\ l)\}_{(\xi,\ l)\in ET\times L} \in \mathbb{R}_+^{|ET\times L|}$ be the consumption of agent $i \in I$,

$c_i(\xi) = \{c_i(\xi,\ l)\}_{l\in L} \in \mathbb{R}_+^{|L|}$ be the vector of consumption of agent $i \in I$ at node $\xi \in ET$,

$p = \{p(\xi,\ l)\}_{(\xi,\ l)\in ET\times L} \in \mathbb{R}^{|ET\times L|}$ be the matrix of spot prices, such that $p(\xi,\ 1) = 1\ \forall \xi \in ET$,

$p(\xi) = \{p(\xi,\ l)\}_{l\in L} \in \mathbb{R}^{|L|}$ be the vector of spot prices at node $\xi \in ET$,

$\tau_d = \{\tau_d(\xi,\ k)\}_{(\xi,\ k)\in ET\times K} \in [0,\ 1]^{|ET\times K|}$ be the matrix of taxes imposed on assets' dividends,

$\tau_d(\xi) = \{\tau_d(\xi,\ k)\}_{k\in K} \in [0,\ 1]^{|K|}$ be the vector of taxes imposed on assets' dividends at node $\xi \in ET$,

$d(\tau_d) = \{d\left(\xi,\ k,\ \tau_d\right)\}_{(\xi,\ k)\in ET\times K} \in \mathbb{R}^{|ET\times K|}$,[45]be the matrix of assets' dividends,

$d(\xi,\ \tau_d) = \{d\left(\xi,\ k,\ \tau_d\right)\}_{k\in K} \in \mathbb{R}^{|K|}$ be the vector of assets' dividends at node $\xi \in ET$,

$z_i = \{z_i(\xi,\ k)\}_{(\xi,\ k)\in ET\times K} \in \mathbb{R}^{|ET\times K|}$ be the asset portfolio held by agent $i \in I$,

$z_i(\xi) = \{z_i(\xi,\ k)\}_{k\in K} \in \mathbb{R}^{|K|}$ be the asset portfolio held by agent $i \in I$ at node $\xi \in ET$,

$q = \{q(\xi,\ k)\}_{(\xi,\ k)\in ET\times K} \in \mathbb{R}^{|ET\times K|}$ be the matrix of asset prices,

$q(\xi) = \{q(\xi,\ k)\}_{k\in K} \in \mathbb{R}^{|K|}$ be the vector of asset prices at node $\xi \in ET$,

$\mathcal{E}(ET,\ (1-\tau_e)\cdot e\left(\tau_e\right),\ \succeq,\ \delta,\ \mathcal{A}(\tau_d))$ be the finite horizon FM Economy with stochastic taxation $\tau = (\tau_e,\ \tau_d) \in [0,1]^{|ET\times L\times I|} \times [0,\ 1]^{|ET\times K|}$ and with assets in strictly positive supply $\delta = \{\delta(k)\}_{k\in K} \in \mathbb{R}_{++}^{|K|}$.[6]

---

[3]For basic notions related to finite horizon FM Economies without stochastic taxation see, for example, Magill and Quinzii (1996), Magill and Shafer (1991) and (1990), Duffie and Shafer (1986). See Appendix A for more definitions related to finite horizon FM Economies with stochastic taxation.

[4]Consistent with the Dividend Clientele Hypothesis (DCH), it is reasonable to assume that individual endowments e_i are decreasing functions $e_i(\tau_{e_i})$ of individual endowment tax rates τ_{e_i} and assets' dividends d are decreasing functions $d(\tau_d)$ of dividend tax rates τ_d. See Kawano (2013), for example, for a review of the DCH. She estimated that a one percentage point decrease in the dividend tax rate relative to the long-term capital gains tax rate leads to a 0.04 percentage point increase in dividend yields. Several papers, including Chetty and Saez (2005), Brown, Liang and Weisbenner (2007) have documented an increase in dividend payments in response to the 2003 tax changes.

[5]In this model, dividends are paid in units of the numeraire good 1. The model could be generalized for the case, where dividends are paid in bundles of all $|L|$ goods. See Duffie and Shafer (1986), for example. In that case $d(\tau_d) = \{d(\xi,\ k,\ l,\ \tau_d)\}_{(\xi,\ k,\ l)\in ET\times K\times L} \in \mathbb{R}^{|ET\times K\times L|}$.

[6]See Appendix A for the definition of an FM economy $\mathcal{E}(ET,\ (1-\tau_e)\cdot e(\tau_e),\ \succeq,\ \delta,\ \mathcal{A}(\tau_d))$ with stochastic taxation $\tau = (\tau_e,\ \tau_d)$ and with assets in strictly positive supply $\delta = \{\delta(k)\}_{k\in K} \in \mathbb{R}_{++}^{|K|}$.

Next, $\forall (q,\, (1-\tau_d)\cdot d(\tau_d)) \in \mathbb{R}^{|ET\times K|}\times \mathbb{R}^{|ET\times K|}$ we will define the Payoff matrix $W(q,\, (1-\tau_d)\cdot d(\tau_d))$, which will significantly simplify writing of agents' budget constraints.[7]

DEFINITION: *$\forall (q,\, (1-\tau_d)\cdot d(\tau_d)) \in \mathbb{R}^{|ET\times K|}\times \mathbb{R}^{|ET\times K|}$ define the $|ET| \times [\,|ET^{-}|\cdot |K|\,]$ Payoff matrix matrix $W(q,\, (1-\tau_d)\cdot d(\tau_d))$ as*

$$W_{\xi,\,\xi^{+}}(q,\, (1-\tau_d)\cdot d(\tau_d)) = q(\xi^{+}) + \left(1-\tau_d(\xi^{+})\right)\cdot d(\xi^{+},\, \tau_d)),$$
$$W_{\xi,\,\xi}(q,\, (1-\tau_d)\cdot d(\tau_d)) = -q(\xi),$$
$$W_{\xi,\,\xi'}(q,\, (1-\tau_d)\cdot d(\tau_d)) = 0\ \forall \xi' \notin \xi^{+},\, \xi' \neq \xi.$$

MATRIX $\mathbf{W}(\mathbf{q}, (\mathbf{1}-\boldsymbol{\tau}_\mathbf{d})\cdot \mathbf{d}(\boldsymbol{\tau}_\mathbf{d}))$

$\lvert K\rvert$ Columns for ξ_0		$\lvert K\rvert$ Columns for ξ^{-}	$\lvert K\rvert$ Columns for ξ		
$-q(\xi_0)$	0	0	0	0	ξ_0
$q(\xi_0^{+})+$ $+\left(1-\tau_d(\xi_0^{+})\right)\cdot$ $\cdot d(\xi_0^{+},\, \tau_d)$	…	…	0	0	ξ_0^{+}
0	…	…	0	0	
0	0	$q(\xi)+$ $+(1-\tau_d(\xi))\cdot$ $\cdot d(\xi,\, \tau_d)$	$-q(\xi)$	0	ξ
0	…	…	0	…	
0	0	0	$q(\xi^{+})+$ $+(1-\tau_d(\xi^{+}))\cdot$ $\cdot d(\xi^{+},\, \tau_d)$	…	ξ^{+}
0	0	0	0	…	

DEFINITION: *Define the budget set of agent $i \in I$ as follows*

$$B(p,\ q,\ (1-\tau_{e_i})\cdot e_i(\tau_{e_i}),\ (1-\tau_d)\cdot d(\tau_d)) =$$
$$\left\{c_i \in \mathbb{R}_{+}^{|ET\times L\times I|}\, |\exists z_i \in \mathcal{Z}\ s.t.\ p\cdot c_i - p\cdot (1-\tau_{e_i})\cdot e_i(\tau_{e_i}) = W(q,\ \ (1-\tau_d)\cdot d(\tau_d))\cdot z_i\right\}.$$

Next, we introduce the notion of an FM equilibrium for an FM economy with stochastic taxation and with assets in strictly positive supply.

DEFINITION: *An FM equilibrium for an FM economy $\mathcal{E}(ET,\, (1-\tau_e)\cdot e(\tau_e),\, \succeq,\, \delta,\, \mathcal{A}(\tau_d))$ with stochastic taxation $\tau = (\tau_e,\, \tau_d) \in [0,1]^{|ET\times L\times I|}\times [0,\ 1]^{|ET\times K|}$ and with assets in strictly positive supply $\delta = \{\delta(k)\}_{k\in K} \in \mathbb{R}_{++}^{|K|}$ is a pair*

$$(\{(\overline{c}_i(\tau),\ \overline{z}_i(\tau))\}_{i\in I}\,,\ \ (\overline{p}(\tau),\ \overline{q}(\tau))) \in \mathbb{R}_{+}^{|ET\times L\times I|}\times \mathcal{Z}^{|I|}\times \mathbb{R}_{+}^{|ET\times L|}\times \mathbb{R}^{|ET\times L|}$$

such that all agents utilities are maximized subject to the budget constraints

$$(\overline{c}_i(\tau),\, \overline{z}_i(\tau)) \in$$
$$\in \arg\max\left\{U_i(c_i,\ G)\mid (c_i,\ z_i) \in B(\overline{p}(\tau),\ \overline{q}(\tau),\ \ (1-\tau_{e_i})\cdot e_i(\tau_{e_i}),\ (1-\tau_d)\cdot d(\tau_d))\right\}\ \forall i \in I,$$

[7] See Magill and Quinzii (1996) for the original definition of the Payoff matrix $W(q,\, d)$ without stochastic taxation.

the equilibrium in commodities markets is given by[8]

$$\sum_{i\in I}\overline{c}_i(\xi,\, l,\, \tau) =$$

$$\begin{cases} \sum_{i\in I}(1-\tau_{e_i}(\xi,\ l))\cdot e_i(\xi,\, l,\, \tau_{e_i}) & \forall\,(\xi,\ l)\in ET\times[L\setminus\{1\}] \\ \sum_{i\in I}(1-\tau_{e_i}(\xi,\ l))\cdot e_i(\xi,\, l,\, \tau_{e_i}) + \sum_{k\in K}(1-\tau_d(\xi,\ k))\cdot d(\xi,\, k,\, \tau_d)\cdot\delta(k) & \forall\,(\xi,\ l)\in ET\times\{1\} \end{cases},$$

the equilibrium in assets markets is given by

$$\sum_{i\in I}\overline{z}_i(\tau) = \delta = \{\delta(k)\}_{k\in K}\in\mathbb{R}^{|K|}_{++}.$$

and the spending on the public good $G = \{G(\xi,\ l)\}_{(\xi,\, l)\in ET\times L}\in\mathbb{R}^{|E\mathbb{T}\times\mathbb{L}|}_{+}$ *is given by*

$$G(\xi,\, l) =$$

$$\begin{cases} \sum_{i\in I}\tau_{e_i}(\xi,\, l)\cdot e_i(\xi,\, l,\ \tau_{e_i}) & \forall(\xi,\, l)\in ET\times[L\setminus\{1\}] \\ \left[\sum_{i\in I}\tau_{e_i}(\xi,\ l)\cdot e_i(\xi,\ l,\ \tau_{e_i}) + \sum_{k\in K}\tau_d(\xi,\ k)\cdot d(\xi,\ k,\ \tau_d)\cdot\delta(k)\right] & \forall(\xi,\, l)\in ET\times\{1\} \end{cases}.$$

DEFINITION: *We define the set of no-arbitrage security prices as*

$$Q = \{q\in\mathbb{R}^{|ET\times K|}\mid \exists\pi\in\mathbb{R}^{|ET|}_{++},\, \pi\cdot W(q,\,(1-\tau_d)\cdot d(\tau_d)) = 0\}.$$

DEFINITION: *Let* $\mathcal{E}(ET,\,(1-\tau_e)\cdot e\,(\tau_e)\,,\,\succeq,\,\delta,\,\mathcal{A}(\tau_d))$ *be an FM economy. Given* $(q,\,(1-\tau_d)\cdot d(\tau_d))\in Q\ \times\mathbb{R}^{|ET\times K|}$, *the FM are complete if* $\exists$ *a ! normalized price vector* $\overline{\pi} = \{\overline{\pi}(\xi')\}_{\xi'\in ET}\in\mathbb{R}^{|ET|}_{++}$ *with* $\overline{\pi}\cdot W(q,\,(1-\tau_d)\cdot d(\tau_d)) = 0$.

2.2. Comparative Statics of FM Equilibria with Respect to the Dividend Tax τ_d

For the rest of this section we will assume

D1 (Assets): *Assets are in strictly positive supply, i.e.,*

$$\sum_{i\in I}z_i = \delta = \{\delta(k)\}_{k\in K}\in\mathbb{R}^{|K|}_{++}.$$

D2 (Preferences): *Agents' preferences* $\succeq_i$ *on* $\mathbb{R}^{|ET\times L|}_{+}\times\mathbb{R}^{|ET\times L|}_{+}$ *are given by the utility function*

$$U_i(c_i,\, G) = \sum_{(\xi,\, l)\in ET\times L}\Pr(\xi)\cdot b_i^{T(\xi)}\cdot[u_i(c_i(\xi,\ l)) + v_i(G(\xi,\ l))],$$

where $u_i\in C^2$ *such that* $u_i'(\cdot) > 0$ *and* $u_i''(\cdot) < 0\ \forall i\in I$.

This form of the utility function makes sense. With exception of the term containing the spending on the public good G

$$\sum_{(\xi,\, l)\in ET\times L}\Pr(\xi)\cdot b_i^{T(\xi)}\cdot v_i(G(\xi,\, l)),$$

the utility function U_i is a special natural case of the utility function from Magill and Quinzii (1994) and (1996)

$$U_i(c_i) = \sum_{\xi\in ET}\Pr(\xi)\cdot b_i^{T(\xi)}\cdot\mathbb{U}_i(c_i(\xi))$$

[8] Again, in this model, dividends are only paid in units of the numeraire good 1. If dividends are paid in bundles of all $|L|$ goods, then equilibrium in commodities markets would be given by $\sum_{i\in I}\overline{c}_i(\xi,\, l,\tau) = \sum_{i\in I}(1-\tau_{e_i}(\xi,\ l))\cdot e_i(\xi,\, l,\tau_{e_i}) + \sum_{k\in K}(1-\tau_d(\xi,\ k,\ l))\cdot d(\xi,\, k,\, l,\, \tau_d)\cdot\delta(k)\ \forall\,(\xi,\ l)\in ET\times L$.

that could be obtained by assuming that

$$\mathbb{U}_i(c_i(\xi)) = \sum_{l\in L} u_i(c_i(\xi,\, l))\ \forall \xi \in ET.$$

D3 (Consumption): *Equilibrium consumption* $\overline{c}_i(1,\,\tau)$ *of the numeraire good* 1 *is differentiable with respect to the dividend tax* τ_d*, such that*

$$\exists \frac{\partial \overline{c}_i(1,\,\tau)}{\partial \tau_d} = \left\{ \left\{ \frac{\partial \overline{c}_i(\xi',\,1,\,\tau)}{\partial \tau_d(\xi,\,k)} \right\}_{k\in K} \right\}_{(\xi',\,\xi)\in ET\times ET} = \left\{ \frac{\partial \overline{c}_i(\xi',\,1,\,\tau)}{\partial \tau_d(\xi)} \right\}_{(\xi',\,\xi)\in ET\times ET}\ \forall i \in I,$$
$$sign\left[\frac{\partial \overline{c}_i(1,\,\tau)}{\partial \tau_d}\right] = sign\left[\frac{\partial \overline{c}_j(1,\,\tau)}{\partial \tau_d}\right]\ \forall (i,\,j) \in I \times I$$

and

$$\frac{\partial \overline{c}_i(\xi',\,1,\,\tau)}{\partial \tau_d\left(\overline{\xi}\right)} = 0\ \forall \left(\overline{\xi},\ i\right) \in \left[ET \setminus ET\left(\xi'\right)\right] \times I,$$

where

$$ET(\xi') = \{\xi \in ET \mid \xi \geqslant \xi'\}.$$

This assumption makes sense. This is a generalization of the traditional case of identical agents, where the equilibrium consumption of the representative agent of the numeraire good 1 at node $\xi' \in ET$

$$\overline{c}_i(\xi',\,1,\,\tau) = (1-\tau_e(\xi',\ 1))\cdot e(\xi',\,1,\,\tau_e) + \sum_{k\in K}(1-\tau_d(\xi',\ k))\cdot d(\xi',\,k,\,\tau_d)\cdot \frac{\delta(k)}{|I|}$$

is unaffected by any dividend tax rate, except $\tau_d\left(\xi'\right)$. In contrast, in this paper we assume that $\overline{c}_i(\xi',\,1,\,\tau)$ is unaffected by any tax rates, except $\tau_d\left(\overline{\xi}\right)\ \forall \overline{\xi} \in ET\left(\xi'\right)$.

D4 (Dividends): *Assets' dividends* d *paid in the units of the numeraire good* 1 *are differentiable with respect to the dividend tax* τ_d*, such that*

$$\frac{\partial d}{\partial \tau_d} = \left\{ \left\{ \frac{\partial d(\xi',\,k_1,\,\tau_d)}{\partial \tau_d(\xi,\,k_2)} \right\}_{(k_1,\,k_2)\in K\times K} \right\}_{(\xi',\,\xi)\in ET\times ET} = \left\{ \frac{\partial d(\xi',\,\tau_d)}{\partial \tau_d(\xi)} \right\}_{(\xi',\,\xi)\in ET\times ET}$$

and

$$\frac{\partial d(\xi',\,k_1,\,\tau_d)}{\partial \tau_d(\xi,\,k_2)} = 0$$

$\forall\left((\xi',\ k_1),\ (\xi,\ k_2)\right) \in [ET \times K] \times [[ET \times K] \setminus \{(\xi',\ k_1)\}]$.

This assumption makes sense. It means that current dividends $d(\xi',\,k,\,\tau_d)$ are unaffected by any tax rates, except $\tau_d\left(\xi',\ k\right)\ \forall\left(\xi',k\right) \in ET \times K$.

D5 (Dividend Tax): *Various dividend tax rates are differentiable with respect to each other, such that*

$$\frac{\partial \tau_d}{\partial \tau_d} = \left\{ \left\{ \frac{\partial \tau_d(\xi',\,k_1)}{\partial \tau_d(\xi,\ k_2)} \right\}_{(k_1,\,k_2,)\in K\times K} \right\}_{(\xi',\,\xi)\in ET\times ET} = \left\{ \frac{\partial \tau_d(\xi',\,\tau_d)}{\partial \tau_d(\xi)} \right\}_{(\xi',\,\xi)\in ET\times ET}$$

and

$$\frac{\partial \tau_d(\xi',\,k_1)}{\partial \tau_d(\xi,\ k_2)} = 0$$

$\forall\left((\xi',\ k_1),\ (\xi,\ k_2)\right) \in [ET \times K] \times [[ET \times K] \setminus \{(\xi',\ k_1)\}]$.

This assumption also makes sense. It means that various dividend tax rates are unaffected by each other.

We start by showing that equilibrium comparative statics with respect to various stochastic tax rates is possible in an open neighborhood of every stochastic tax rate for which an FM equilibrium exists.

DEFINITION: *An FM equilibrium*

$$\left(\left\{(\overline{c}_i,\ \overline{z}_i)\right\}_{i\in I},\ (\overline{p},\ \overline{q})\right) \in \left(\mathbb{R}_{+}^{|ET\times L\times I|} \times \mathcal{Z}^{|I|}\right) \times \left(\mathbb{R}_{++}^{|ET\times L|} \times Q\ \right)$$

for an FM economy $\mathcal{E}(ET,\ (1-\tau_e)\cdot e(\tau_e),\ \succeq,\ \delta,\ \mathcal{A}(\tau_d))$ *is called regular if*

$$det\left[D_{(p,\ q)}ED(\overline{p},\ \overline{q},\ \ (1-\tau_e)\cdot e(\tau_e),\ (1-\tau_d)\cdot d(\tau_d))\right] \neq 0,$$

where $D_{(p,\ q)}ED$ *is the Jacobian of the excess demand function* ED.

LEMMA 2.2.1: *Suppose Assumptions D1-D5 hold. Let*

$$\left(\left\{(\overline{c}_i,\ \overline{z}_i)\right\}_{i\in I},\ (\overline{p},\ \overline{q})\right) \in \left(\mathbb{R}_{+}^{|ET\times L\times I|} \times \mathcal{Z}^{|I|}\right) \times \left(\mathbb{R}_{++}^{|ET\times L|} \times Q\ \right)$$

be a regular FM equilibrium in which markets are complete for an FM economy $\mathcal{E}(ET,\ (1-\overline{\tau}_e)\cdot e(\overline{\tau}_e),$ $\succeq,\ \delta,\ \mathcal{A}(\overline{\tau}_d))$. *Then* $\exists$ *an open neighborhood* $O_{\overline{\tau}} \subset [0,\ 1]^{|ET\times L\times I|} \times [0,\ 1]^{|ET\times K|}$ *of* $\overline{\tau}$ *and a function*

$$h : O_{\overline{\tau}} \longrightarrow \left(\mathbb{R}_{+}^{|ET\times L\times I|} \times \mathcal{Z}^{|I|}\right) \times \left(\mathbb{R}_{++}^{|ET\times L|} \times Q\ \right)$$

defined as

$$h(\tau) = \left(\left\{(\overline{c}_i(\tau),\ \overline{z}_i(\tau))\right\}_{i\in I},\ (\overline{p}(\tau),\ \overline{q}(\tau))\right),$$

where

$$\left(\left\{(\overline{c}_i(\tau),\ \overline{z}_i(\tau))\right\}_{i\in I},\ \ (\overline{p}(\tau),\ \overline{q}(\tau))\right) \in \left(\mathbb{R}_{+}^{|ET\times L\times I|} \times \mathcal{Z}^{|I|}\right) \times \left(\mathbb{R}_{++}^{|ET\times L|} \times Q\ \right)$$

be an FM equilibrium in which markets are complete for an FM economy $\mathcal{E}(ET,\ (1-\tau_e)\cdot e(\tau_e),$ $\succeq,\ \delta,\ \mathcal{A}(\tau_d))$, *i.e.,* $\exists$ *a ! normalized price vector* $\overline{\pi}(\tau) = \{\overline{\pi}(\xi',\ \tau)\}_{\xi'\in ET} \in \mathbb{R}_{++}^{|ET|}$ *with* $\overline{\pi}(\tau)\cdot W(\overline{q}(\tau),$ $(1-\tau_d)\cdot d(\tau_d)) = 0\ \forall\tau \in O_{\overline{\tau}}$. *Moreover,* $h \in C^1\left(O_{\overline{\tau}}\right)$.

PROOF: See Appendix B.

DEFINITION: *Define function*

$$H : O_{\overline{\tau}} \longrightarrow \left(\mathbb{R}_{+}^{|ET\times L\times I|} \times \mathcal{Z}^{|I|}\right) \times \left(\mathbb{R}_{++}^{|ET\times L|} \times Q\ \right) \times \mathbb{R}_{++}^{|ET|}$$

as follows

$$H(\tau) = \left(\left\{(\overline{c}_i(\tau),\ \overline{z}_i(\tau))\right\}_{i\in I},\ (\overline{p}(\tau),\ \overline{q}(\tau)),\ \overline{\pi}(\tau)\right)\ \forall\tau \in O_{\overline{\tau}},$$

where

$$\left(\left\{(\overline{c}_i(\tau),\ \overline{z}_i(\tau))\right\}_{i\in I},\ \ (\overline{p}(\tau),\ \overline{q}(\tau))\right) \in \left(\mathbb{R}_{+}^{|ET\times L\times I|} \times \mathcal{Z}^{|I|}\right) \times \left(\mathbb{R}_{++}^{|ET\times L|} \times Q\ \right)$$

be an FM equilibrium in which markets are complete for an FM economy $\mathcal{E}(ET,\ (1-\tau_e)\cdot e(\tau_e),$ $\succeq,\ \delta,\ \mathcal{A}(\tau_d))$, *s.t.* $\exists$ *a ! normalized price vector* $\overline{\pi}(\tau) = \{\overline{\pi}(\xi',\ \tau)\}_{\xi'\in ET} \in \mathbb{R}_{++}^{|ET|}$ *with* $\overline{\pi}(\tau)\cdot W(\overline{q}(\tau),$ $(1-\tau_d)\cdot d(\tau_d)) = 0$.

Let us first analyze how a change in the current dividend tax rate $\tau_d(\xi) \in \mathbb{R}^{|K|}$ will affect current equilibrium asset prices $\overline{q}(\xi,\ \tau) \in \mathbb{R}^{|K|}$. Since a change in $\tau_d(\xi)$ might affect various node prices $\overline{\pi}(\xi',\ \tau)$ and after-tax dividends $(1-\tau_d(\xi')) \cdot d(\xi',\ \tau_d)$, $\xi' \in ET^{+}(\xi)$ differently, the net effect of $\tau_d(\xi)$ on $\overline{q}(\xi,\ \tau)$ is ambiguous. We need to impose Assumptions D1-D5 from above to remove this ambiguity. We will be able to derive economically intuitive comparative statics of $\overline{q}(\xi,\ \tau)$ with respect to $\tau_d(\xi)$ results without assuming either CRRA utility functions or identical agents. We will start our analysis with the following lemma:

LEMMA 2.2.2: *Suppose assumptions of the above Lemma 2.2.1. hold. Let*

$$\left(\{(\overline{c}_i,\ \overline{z}_i)\}_{i\in I},\ (\overline{p},\ \overline{q})\right) \in \left(\mathbb{R}_{+}^{|ET\times L\times I|} \times \mathcal{Z}^{|I|}\right) \times \left(\mathbb{R}_{++}^{|ET\times L|} \times Q\right)$$

be a regular FM equilibrium in which markets are complete for an FM economy $\mathcal{E}(ET,\ (1-\overline{\tau}_e)\cdot e(\overline{\tau}_e),\ \succeq,\ \delta,\ \mathcal{A}(\overline{\tau}_d))$*, Let* ξ *be the initial node of the event tree* ET*. Then an open neighborhood* $O_{\overline{\tau}}$ *of* $\overline{\tau}$ *and a function*

$$H(\tau) = \left(\{(\overline{c}_i(\tau),\ \overline{z}_i(\tau))\}_{i\in I},\ \ (\overline{p}(\tau),\ \overline{q}(\tau)),\ \overline{\pi}(\tau)\right)\ \forall \tau \in O_{\overline{\tau}},$$

obtained in the previous Lemma 2.2.1. are s.t.

$$\boxed{\overline{\pi}(\xi',\ \tau) = b_i^{T(\xi')} \cdot \frac{u_i'(\overline{c}_i(\xi',\ 1,\ \tau))}{u_i'(\overline{c}_i(\xi,\ 1,\ \tau))} \cdot \Pr(\xi')\ \forall (\xi',\ i) \in ET^{+}(\xi) \times I,} \tag{1}$$

$$\boxed{\overline{q}(\xi,\ \tau) = \sum_{\xi' \in ET^{+}(\xi)} b_i^{T(\xi')} \cdot \frac{u_i'(\overline{c}_i(\xi',\ 1,\ \tau))}{u_i'(\overline{c}_i(\xi,\ 1,\ \tau))} \cdot \Pr(\xi') \cdot (1-\tau_d(\xi')) \cdot d(\xi',\ \tau_d)} \tag{2}$$

and

$$\boxed{\frac{\partial \overline{\pi}(\xi',\ \tau)}{\partial \tau_d(\overline{\xi})} = \overline{\pi}(\xi',\tau) \cdot \left[rr_i(\overline{c}_i(\xi,\ 1,\ \tau)) \cdot g_{\overline{c}_i(\xi,\ 1,\ \tau)} - rr_i(\overline{c}_i(\xi',\ 1,\ \tau)) \cdot g_{\overline{c}_i(\xi',\ 1,\ \tau)}\right]\ \forall \overline{\xi} \in ET,} \tag{3}$$

where

$$\frac{\partial \overline{\pi}}{\partial \tau_d} = \left\{\left\{\frac{\partial \overline{\pi}(\xi',\ \tau)}{\partial \tau_d(\xi,\ k)}\right\}_{k\in K}\right\}_{(\xi',\ \xi)\in ET\times ET} = \left\{\frac{\partial \overline{\pi}(\xi',\ \tau)}{\partial \tau_d(\xi)}\right\}_{(\xi',\ \xi)\in ET\times ET},$$

$$rr_i(c) = -\left[\frac{u_i''(c)\cdot c}{u_i'(c)}\right]$$

is the coefficient of relative risk aversion of an agent $i \in I$ *and*

$$g_{\overline{c}_i(\xi',\ 1,\ \tau)} = \frac{1}{\overline{c}_i(\xi',\ 1,\ \tau)} \cdot \frac{\partial \overline{c}_i(\xi',\ 1,\ \tau)}{\partial \tau_d(\overline{\xi})}\ \forall \xi' \in ET.$$

PROOF: See Appendix B.

We will use the Lemma 2.2.2. to determine the sign of

$$\frac{\partial \overline{q}(\xi,\ \tau)}{\partial \tau_d(\xi)} = \left\{\frac{\partial \overline{q}(\xi,\ k_1,\ \tau)}{\partial \tau_d(\xi,\ k_2)}\right\}_{(k_1,\ k_2)\in K\times K} \in \mathbb{R}^{|K\times K|}\ \forall \xi \in ET.$$

THEOREM 2.2.3: *Let*

$$\left(\{(\overline{c}_i,\ \overline{z}_i)\}_{i\in I},\ (\overline{p},\ \overline{q})\right) \in \left(\mathbb{R}_+^{|ET\times L\times I|} \times \mathcal{Z}^{|I|}\right) \times \left(\mathbb{R}_+^{|ET\times L|} \times Q\right)$$

be a regular FM equilibrium in which markets are complete for an FM economy $\mathcal{E}(ET,\ (1-\overline{\tau}_e)\cdot e(\overline{\tau}_e),\ \succeq,\ \delta,\ \mathcal{A}(\overline{\tau}_d))$. Let ξ be the initial node of the event tree ET. Then an open neighborhood $O_{\overline{\tau}}$ of $\overline{\tau}$ and a function

$$H(\tau) = \left(\{(\overline{c}_i(\tau),\ \overline{z}_i(\tau))\}_{i\in I},\ \ (\overline{p}(\tau),\ \overline{q}(\tau)),\ \overline{\pi}(\tau)\right)\ \forall\tau\in O_{\overline{\tau}},$$

obtained in the previous Lemma 2.2.1. are s.t.

$$\boxed{\frac{\partial\overline{q}(\xi,\ \tau)}{\partial\tau_d(\xi)} = \overline{q}\,(\xi,\ \tau)\cdot rr_i(\overline{c}_i\,(\xi,\ 1,\ \tau))\cdot g_{\overline{c}_i(\xi,\ 1,\ \tau)}} \tag{4}$$

and

$$\boxed{sign\ \boxed{\frac{\partial\overline{q}(\xi,\ \tau)}{\partial\tau_d(\xi)}} = sign\ \boxed{\frac{\partial\overline{c}_i(\xi,\ 1,\ \tau)}{\partial\tau_d(\xi)}}\ \forall\,(\xi,\ i)\in ET\times I,}$$

where

$$g_{\overline{c}_i(\xi,\ 1,\ \tau)} = \frac{1}{\overline{c}_i(\xi,\ 1,\ \tau)}\cdot\frac{\partial\overline{c}_i(\xi,\ 1,\ \tau)}{\partial\tau_d(\xi)}.$$

PROOF: See Appendix B.

The economic interpretation of the above result is as follows. Note first that an increase in $\tau_d\,(\xi)$ affects $\overline{q}\,(\xi,\ \tau)$ only through stochastic discount factors $\overline{\pi}(\xi',\ \tau),\ \xi'\in ET^+(\xi)$. Although the sign of a derivative of current equilibrium asset prices $\overline{q}\,(\xi,\ \tau)$ with respect to current dividend tax rates $\tau_d\,(\xi)$ may be ambiguous, it is always the same as that of the derivative of the current equilibrium consumption $\overline{c}_i\,(\xi,\ 1,\ \tau)$ of the numeraire good 1 with respect to current dividend tax rates $\tau_d\,(\xi)$. Depending on the sign of $\frac{\partial\overline{c}_i(\xi,\ 1,\ \tau)}{\partial\tau_d(\xi)}$, an increase in $\tau_d\,(\xi)$ might be reducing or boosting $\overline{q}\,(\xi,\ \tau)$. Therefore, we have two cases to consider here:

Suppose first $\frac{\partial\overline{c}_i(\xi,\ 1,\ \tau)}{\partial\tau_d(\xi)} \leqslant 0$, i.e., the numeraire good 1 is a normal good. Then an increase in $\tau_d\,(\xi)$ reduces $\overline{\pi}(\xi',\ \tau)$, thus decreasing today's price of the future consumption $\overline{c}_i\,(\xi',\ 1,\ \tau),\ \xi'\in ET^+(\xi)$ of the numeraire good 1. Since financial assets represent claims on future consumption, the increase in $\tau_d\,(\xi)$ reduces today's asset prices $\overline{q}\,(\xi,\ \tau)$.

Suppose now $\frac{\partial\overline{c}_i(\xi,\ 1,\ \tau)}{\partial\tau_d(\xi)} > 0$, i.e., the numeraire good 1 is an inferior good. Then an increase in $\tau_d\,(\xi)$ boosts $\overline{\pi}(\xi',\ \tau)$, thus increasing today's price of the future consumption $\overline{c}_i\,(\xi',\ 1,\ \tau),\ \xi'\in ET^+(\xi)$ of the numeraire good 1. Since financial assets represent claims on future consumption, the increase in $\tau_d\,(\xi)$ boosts today's asset prices $\overline{q}\,(\xi,\ \tau)$.

While inferior goods have been demonstrated to exist, it is generally believed that they are rare: at any given time and price level, the demand for the vast majority of goods moves in the intuitive direction with respect to changes in after-tax income. So it is reasonable to assume that the numeraire good 1 is a normal good. Therefore, it is natural to conclude that under reasonable assumptions, an increase in the current dividend tax rate reduces current asset prices.

COROLLARY 2.2.4: *Suppose assumptions of the above Theorem 2.2.3. hold. Assume further that all agents are identical and exhibit CRRA, i.e.,*

$$rr_i(c) = -\left[\frac{u''(c)\cdot c}{u'(c)}\right] = a\ \forall i\in I.$$

Assets' dividends are taxed identically, i.e.,

$$\tau_d(\xi,\ k) = \tau_d(\xi,\ \overline{k})\ \forall\,(\xi,\ k)\in ET\times K.$$

In addition, agents have zero initial endowments of the numeraire good 1, i.e.,

$$e_i(\xi,\ 1,\ \tau) = 0\ \forall\,(i,\ \xi) \in I \times ET^9$$

and

$$\frac{\partial d}{\partial \tau_d} = 0.$$

Then

$$\boxed{E_{q(\xi,\ \tau),\ 1-\tau_d(\xi,\ \overline{k})} = a,}$$

where

$$E_{q(\xi,\ \tau),\ 1-\tau_d(\xi,\ \overline{k})} = \frac{\frac{1}{\overline{q}(\xi,\ \tau)}\frac{\partial \overline{q}(\xi,\ \tau)}{\partial \tau_d(\xi,\ \overline{k})}}{\frac{1}{(1-\tau_d(\xi,\ \overline{k}))}\frac{\partial(1-\tau_d(\xi,\ \overline{k}))}{\partial \tau_d(\xi,\ \overline{k})}}\ \forall \xi \in ET$$

is the elasticity of asset prices $q(\xi,\ \tau)$ with respect to the economic freedom $1 - \tau_d\left(\xi,\ \overline{k}\right)$ at a node $\xi \in ET$.

PROOF: See Appendix B.

The expression

$$\sum_{k \in K} d(\xi,\ k,\ \tau_d) \cdot \delta(k)$$

can be interpreted as the country's total GDP at node $\xi \in ET$ and

$$(1 - \tau_d(\xi,\ \overline{k})) = \frac{\sum_{k \in K}(1-\tau_d(\xi,\ \overline{k}))\cdot d(\xi,\ k,\ \tau_d)\cdot\delta(k)}{\sum_{k \in K} d(\xi,\ k,\ \tau_d)\cdot\delta(k)}$$

can be interpreted as a percentage of the economy's total GDP consumed by the private sector at a node $\xi \in ET$. Hence, $(1 - \tau_d(\xi,\ \overline{k}))$ can be interpreted as the economy's level of economic freedom at a node $\xi \in ET$. Numerically, Magin (2015a) estimated the coefficient of agents' relative risk aversion, $a = 3.76$. So on average, for S&P 500 stocks, a 1% increase of the economy's economic freedom generates a 3.76% increase in share prices.

COROLLARY 2.2.5: *Suppose assumptions of Theorem 2.2.3. hold. Then*

$$\boxed{sign\ \boxed{\frac{\partial R(\xi',\ \tau)}{\partial \tau_d(\xi)}} = sign\ \boxed{\frac{\partial ATR(\xi',\ \tau)}{\partial \tau_d(\xi)}} = -sign\ \boxed{\frac{\partial q(\xi,\ \tau)}{\partial \tau_d(\xi)}}\ \forall\,(\xi,\ \xi') \in ET \times \xi^+,}$$

where

$$R(\xi',\ \tau) = \frac{q(\xi',\ \tau)+d(\xi',\ \tau_d)}{q(\xi,\ \tau)}$$

is the total before-tax rate of return of an asset at a node $\xi' \in \xi^+$ and

$$ATR(\xi',\ \tau) = \frac{q(\xi',\ \tau)+(1-\tau_d(\xi'))\cdot d(\xi',\ \tau_d)}{q(\xi,\ \tau)}$$

is the total after-tax rate of return of an asset at a node $\xi' \in \xi^+$.

PROOF: See Appendix B.

Empirical findings of Sialm (2006) and (2009) imply that stocks with heavier tax burdens tend to compensate taxable investors by offering higher before-tax returns and equity premia. Corollary 2.2.5. demonstrates that this is not necessarily the case.

[9] We can obtain a similar result without assuming zero initial endowments. It is sufficient to assume $\tau_d(\xi) = \tau_e(\xi)\ \forall \xi \in ET$ instead.

Let us now analyze how a change in a future $\tau_d\left(\overline{\xi}\right)$, $\overline{\xi} \in ET^+(\xi)$ stochastic dividend tax rate will affect current equilibrium asset prices $\overline{q}(\xi,\ \tau)$. Again, since a change in $\tau_d\left(\overline{\xi}\right)$ might affect various node prices $\overline{\pi}(\xi',\tau)$, $\xi' \in ET^+(\xi)$ and after-tax dividends $(1-\tau_d(\xi'))\cdot d(\xi',\ \tau_d)$, $\xi' \in ET^+(\xi)$ differently, the net effect of $\tau_d\left(\overline{\xi}\right)$ on $\overline{q}(\xi,\ \tau)$ is ambiguous. Unlike the comparative statics of $\overline{q}(\xi,\ \tau)$ with respect to $\tau_d(\xi)$, it does not appear to be possible to derive economically intuitive comparative statics of $\overline{q}(\xi,\ \tau)$ with respect to $\tau_d\left(\overline{\xi}\right)$ results without assuming either CRRA utility functions or identical agents.

Suppose now that agents exhibit CRRA but are not necessarily identical. Our goal here is to determine the sign of

$$\frac{\partial \overline{q}(\xi,\ \tau)}{\partial \tau_d\left(\overline{\xi}\right)} = \left\{\frac{\partial \overline{q}(\xi,\ k_1,\ \tau)}{\partial \tau_d\left(\overline{\xi},\ k_2\right)}\right\}_{(k_1,\ k_2)\in K\times K} \in \mathbb{R}^{|K\times K|}\ \forall \overline{\xi} \in ET^+(\xi).$$

THEOREM 2.2.6: *Let*

$$\left(\{(\overline{c}_i,\ \overline{z}_i)\}_{i\in I},\ (\overline{p},\ \overline{q})\right) \in \left(\mathbb{R}_+^{|ET\times L\times I|} \times \mathcal{Z}^{|I|}\right) \times \left(\mathbb{R}_+^{|ET\times L|} \times Q\right)$$

be a regular FM equilibrium in which markets are complete for an FM economy $\mathcal{E}(ET,\ (1-\overline{\tau}_e)\cdot e(\overline{\tau}_e),\ \succeq,\ \delta,\ \mathcal{A}(\overline{\tau}_d))$, *s.t. agents' preferences* $\succeq_i$ *on* $\mathbb{R}_+^{|ET\times L|} \times \mathbb{R}_+^{|ET\times L|}$ *are given by the utility function*

$$U_i(c_i,\ G) = \sum_{(\xi,\ l)\in ET\times L} \Pr(\xi)\cdot b_i^{T(\xi)} \cdot [u_i(c_i(\xi,\ l)) + v_i(G(\xi,\ l))]\ \forall i \in I,$$

where u_i *is a CRRA utility function such that* $u_i(c) = \frac{c^{1-a}}{1-a}\ \forall i \in I$.
Assume further that an open neighborhood $O_{\overline{\tau}}$ *of* $\overline{\tau}$ *and a function*

$$H(\tau) = \left(\{(\overline{c}_i(\tau),\ \overline{z}_i(\tau))\}_{i\in I},\ \left(\overline{p}(\tau),\ \overline{q}(\tau)\right),\ \overline{\pi}(\tau)\right)\ \forall \tau \in O_{\overline{\tau}},$$

obtained in the previous Lemma 2.2.1. are s.t.

$$\frac{\overline{c}_i(\xi',\ 1,\ \tau)}{\overline{c}_i(\xi,\ 1,\ \tau)} = \frac{\sum_{i\in I}\left(1-\tau_{e_i}(\xi',\ 1)\right)\cdot e_i(\xi',\ 1,\ \tau_{e_i}) + \sum_{k\in K}(1-\tau_d(\xi',\ k))\cdot d(\xi',\ k,\ \tau_d)\cdot\delta(k)}{\sum_{i\in I}\left(1-\tau_{e_i}(\xi,\ 1)\right)\cdot e_i(\xi,\ 1,\ \tau_{e_i}) + \sum_{k\in K}(1-\tau_d(\xi,\ k))\cdot d(\xi,\ k,\ \tau_d)\cdot\delta(k)}\text{[10]}$$

$\forall\,(\xi,\ \xi',\ \tau,\ i) \in ET \times ET \times O_{\overline{\tau}} \times I$, *where* $\delta(k)$ *is the total number of outstanding shares of an asset* $k \in K$. *Let* ξ *be the initial node of the event tree* ET. *Fix* $\overline{\xi} \in ET^+(\xi)$. *Then*

$$\boxed{sign\ \boxed{\frac{\partial \overline{q}(\xi,\ k,\ \tau)}{\partial \tau_d\left(\overline{\xi},\ k\right)}} = sign\left[a - B(\overline{\xi},\ k,\ \tau)\right]\ \forall\left(\overline{\xi},\ k\right) \in ET^+(\xi)\times K,\text{[11]}}$$

where

$$B(\overline{\xi},\ k,\ \tau) = \frac{\frac{1}{(1-\tau_d(\overline{\xi},\ k))\cdot d(\overline{\xi},\ k,\ \tau_d)\cdot\delta(k)}}{\sum_{i\in I}(1-\tau_{e_i}(\overline{\xi},\ 1))\cdot e_i(\overline{\xi},\ 1,\ \tau_{e_i}) + \sum_{k\in K}(1-\tau_d(\overline{\xi},\ k))\cdot d(\overline{\xi},\ k,\ \tau_d)\cdot\delta(k)} \geqslant 1$$

$\forall\left(\overline{\xi},\ k\right) \in ET^+(\xi)\times K$.

PROOF: See Appendix B.

Moreover, if $|K| = 1$ and $e_i(\overline{\xi},\ 1,\ \tau_{e_i}) = 0\ \forall i \in I$, then $B(\overline{\xi},\ k,\ \tau) = 1$.

---

[10] This assumption makes sense. It means that agents' consumption of the numeraire good 1 is growing at the same rate as the total after-tax output of the numeraire good 1.

[11] If $k_1 \neq k_2$, then $\frac{\partial \overline{q}(\xi,\ k_1,\ \tau)}{\partial \tau_d\left(\overline{\xi},\ k_2\right)} = a\cdot\overline{\pi}(\overline{\xi},\tau)\cdot\Pi(\overline{\xi},\ k,\ \tau)\cdot\left(1-\tau_d\left(\overline{\xi},\ k\right)\right)\cdot d\left(\overline{\xi},\ k,\ \tau_d\right) > 0.$

The economic interpretation of the above result is as follows. Fix $\overline{\xi} \in ET^{+}(\xi)$. Note first that an increase in $\tau_d\left(\overline{\xi},\ k\right)$ affects $\overline{q}\left(\xi,\ k,\ \tau\right)$ through both stochastic discount factors $\overline{\pi}(\xi',\ \tau)$ and after-tax dividends $\left(1-\tau_d\left(\xi',\ k\right)\right)\cdot d\left(\xi',\ k\ ,\tau_d\right)$, $\xi' \in ET^{+}(\xi)$. Under the assumptions of the theorem, an increase in $\tau_d\left(\overline{\xi},\ k\right)$ generates two effects which are working in opposite directions. On the one hand, an increase in $\tau_d\left(\overline{\xi},\ k\right)$ boosts the stochastic discount factor $\overline{\pi}(\overline{\xi},\tau)$, thus boosting asset prices $\overline{q}\left(\xi,\ k,\ \tau\right)$. On the other hand, an increase in $\tau_d\left(\overline{\xi},\ k\right)$ reduces after-tax dividends $\left(1-\tau_d\left(\overline{\xi},\ k\right)\right)\cdot d\left(\overline{\xi},\ k,\ \tau_d\right)$, thus reducing asset prices $\overline{q}\left(\xi,\ k,\ \tau\right)$. The net effect of the increase in $\tau_d\left(\overline{\xi},\ k\right)$ on $\overline{q}\left(\xi,\ k,\ \tau\right)$ is ambiguous and is determined by the value of the coefficient of relative risk aversion a. We have three cases to consider here:

If $a > B(\overline{\xi}, k, \tau)$, i.e., the coefficient of relative risk aversion a is high, then an increase in $\tau_d\left(\overline{\xi},\ k\right)$ generates such a strong boosting effect on the stochastic discount factor $\overline{\pi}(\overline{\xi},\tau)$ that it dominates the reducing effect it has on after-tax dividends $\left(1-\tau_d\left(\overline{\xi},\ k\right)\right)\cdot d\left(\overline{\xi},\ k,\ \tau_d\right)$, thus boosting asset prices $\overline{q}\left(\xi,\ k,\ \tau\right)$.

If $a < B(\overline{\xi},\ k,\ \tau)$, i.e., the coefficient of relative risk aversion a is low, then an increase in $\tau_d\left(\overline{\xi},\ k\right)$ generates such a weak boosting effect on the stochastic discount factor $\overline{\pi}(\overline{\xi},\ \tau)$ that it is dominated by the reducing effect it has on after-tax dividends $\left(1-\tau_d\left(\overline{\xi},\ k\right)\right)\cdot d\left(\overline{\xi},\ k,\ \tau_d\right)$, thus reducing asset prices $\overline{q}\left(\xi,\ k,\ \tau\right)$.

Finally, if $a = B(\overline{\xi}, k, \tau)$, then an increase in $\tau_d\left(\overline{\xi},\ k\right)$ generates such a boosting effect on the stochastic discount factor $\overline{\pi}(\overline{\xi},\tau)$ that it is completely canceled out by the reducing effect it has on after-tax dividends $\left(1-\tau_d\left(\overline{\xi},\ k\right)\right)\cdot d\left(\overline{\xi},\ k,\ \tau_d\right)$, thus leaving asset prices $\overline{q}\left(\xi,\ k,\ \tau\right)$ unchanged.

Suppose agents are identical but do not necessarily exhibit CRRA.

THEOREM 2.2.7: *Suppose assumptions of the above Theorem 2.2.6. hold, except that now identical agents' preferences* $\succeq_i$ *on* $\mathbb{R}_{+}^{|ET\times L|}\times\mathbb{R}_{+}^{|ET\times L|}$ *are given by the utility function*

$$U_i(c,\,G)=\sum_{(\xi,\,l)\in ET\times L}\Pr(\xi)\cdot b^{T(\xi)}\cdot\left[u(c(\xi,\ l))+v(G(\xi,\ l))\right]\ \forall i\in I,$$

where $u\in C^2$ *such that* $u'(\cdot)>0$ *and* $u"(\cdot)<0$. *Assume further that*

$$sign\left[\frac{\partial d(\xi',\,k,\,\tau_d)}{\partial\tau_d\left(\overline{\xi},\,k\right)}\right]=-1\ for\ \overline{\xi}=\xi'.$$

Let ξ *be the initial node of the event tree* ET. *Fix* $\overline{\xi}\in ET^{+}(\xi)$. *Then*

$$\boxed{sign\left[\frac{\partial\overline{q}(\xi,\,k,\,\tau)}{\partial\tau_d\left(\overline{\xi},\,k\right)}\right]=sign\left[rr(\overline{c}(\overline{\xi},\ 1,\ \tau))-B(\overline{\xi},\ k,\ \tau)\right],}$$

where

$$rr(c)=-\left[\frac{u"(c)\cdot c}{u'(c)}\right]$$

is the coefficient of agents' relative risk aversion and

$$B(\overline{\xi},\,k,\,\tau)=\frac{\frac{1}{\left(1-\tau_d\left(\overline{\xi},\,k\right)\right)\cdot d\left(\overline{\xi},\,k,\,\tau_d\right)\cdot\delta(k)}}{\sum_{i\in I}\left(1-\tau_{e_i}\left(\overline{\xi},\,1\right)\right)\cdot e_i\left(\overline{\xi},\,1,\,\tau_{e_i}\right)+\sum_{k\in K}\left(1-\tau_d\left(\overline{\xi},\,k\right)\right)\cdot d\left(\overline{\xi},\,k,\,\tau_d\right)\cdot\delta(k)}\gtrless 1$$

$\forall\left(\overline{\xi},\ k\right)\in ET^{+}(\xi)\times K$.

PROOF: See Appendix B.

COROLLARY 2.2.8: *Suppose assumptions of the above Theorem 2.2.7. hold, except that now*

$$\frac{\partial d}{\partial \tau_d} = 0.$$

Assume further that identical agents have zero initial endowments of the numeraire good 1, i.e.,

$$e_i(\xi,\, 1,\, \tau) = 0 \;\forall\, (i,\; \xi) \in I \times ET$$

and all assets' dividends are taxed identically, i.e.,

$$\tau_d(\xi,\, k) = \tau_d(\xi,\, \overline{k}) \;\forall\, (\xi,\; k) \in ET \times K.$$

Then

$$\boxed{sign \boxed{\frac{\partial \overline{q}(\xi,\, k,\, \tau)}{\partial \tau_d\left(\xi,\, \overline{k}\right)}} = sign\left[rr(\overline{c}(\overline{\xi},\; 1,\; \tau)) - 1\right] \;\forall \overline{\xi} \in ET^{+}(\xi).}$$

PROOF: See Appendix B.

2.3. Comparative Statics of FM Equilibria with Respect to the Endowment Tax τ_{e_i}

For the rest of this section we will assume

E1 (Assets): *Assets are in positive supply, i.e.,*

$$\sum_{i \in I} z_i = \delta = \{\delta(k)\}_{k \in K} \in \mathbb{R}^{|K|}_{++}.$$

E2 (Preferences): *Agents' preferences $\succeq_i$ on $\mathbb{R}^{|ET \times L|}_{+} \times \mathbb{R}^{|ET \times L|}_{+}$ are given by the utility function*

$$U_i(c_i,\, G) = \sum_{(\xi,\, l) \in ET \times L} \Pr(\xi) \cdot b_i^{T(\xi)} \cdot [u_i(c_i(\xi,\; l)) + v_i(G(\xi,\; l))],$$

where $u_i \in C^2$ such that $u_i'(\cdot) > 0$ and $u_i"(\cdot) < 0 \;\forall i \in I$.

E3 (Consumption): *Equilibrium consumption $\overline{c}_i(1,\, \tau)$ of the numeraire good 1 is differentiable with respect to the endowment tax τ_{e_i}, such that*

$$\frac{\partial \overline{c}_i(1,\, \tau)}{\partial \tau_{e_i}} = \left\{ \left\{ \frac{\partial \overline{c}_i(\xi',\, 1,\, \tau)}{\partial \tau_{e_i}(\xi,\, l)} \right\}_{l \in L} \right\}_{(\xi',\, \xi) \in ET \times ET} = \left\{ \frac{\partial \overline{c}_i(\xi',\, 1,\, \tau)}{\partial \tau_{e_i}(\xi)} \right\}_{(\xi',\, \xi) \in ET \times ET}$$

and

$$sign\left[\frac{\partial \overline{c}_i(1,\, \tau)}{\partial \tau_{e_i}}\right] = sign\left[\frac{\partial \overline{c}_j(1,\, \tau)}{\partial \tau_{e_i}}\right] \;\forall (i,\, j) \in I \times I$$

and

$$\frac{\partial \overline{c}_i(\xi',\, 1,\, \tau)}{\partial \tau_{e_i}\left(\overline{\xi}\right)} = 0 \;\forall \overline{\xi} \in ET \setminus ET\,(\xi').$$

This assumption makes sense. This is a generalization of the traditional case of identical agents, where equilibrium consumption of the representative agent of the numeraire good 1 at node $\xi' \in ET$

$$\overline{c}_i(\xi',\, 1,\, \tau) = (1 - \tau_e(\xi',\; 1)) \cdot e(\xi',\, 1,\, \tau_e) + \sum_{k \in K} (1 - \tau_d(\xi',\; k)) \cdot \frac{\delta(k)}{|I|} \cdot d(\xi',\, k,\, \tau_d)$$

is unaffected by any endowment tax rate, except $\tau_e(\xi')$. In contrast, in this paper we assume that $\overline{c}_i(\xi',$ $1,\ \tau)$ is unaffected by any tax rates, except $\tau_e\left(\overline{\xi}\right)\ \forall\overline{\xi}\in ET\left(\xi'\right)$.

E4 (Dividends): *Assets' dividends d paid in the units of the numeraire good 1 are differentiable with respect to the endowment tax τ_{e_i}, such that*

$$\frac{\partial d}{\partial\tau_{e_i}}=\left\{\left\{\frac{\partial d(\xi',\,k,\,\tau_d)}{\partial\tau_{e_i}(\xi,\,l)}\right\}_{(k,\,l)\in K\times L}\right\}_{(\xi',\,\xi)\in ET\times ET}=\left\{\frac{\partial d(\xi',\,\tau_d)}{\partial\tau_{e_i}(\xi)}\right\}_{(\xi',\,\xi)\in ET\times ET}=0.$$

This assumption also makes sense. It means that dividends d are unaffected by various endowment tax rates τ_{e_i}.

E5 (Endowment Tax): *Various dividend tax rates τ_d are differentiable with respect to various endowment tax rates τ_{e_i}, such that*

$$\frac{\partial\tau_d}{\partial\tau_{e_i}}=\left\{\left\{\frac{\partial\tau_d(\xi',\,k,\,\tau_d)}{\partial\tau_{e_i}(\xi,\,l)}\right\}_{(k,\,l,)\in K\times L}\right\}_{(\xi',\,\xi)\in ET\times ET}=\left\{\frac{\partial\tau_d(\xi')}{\partial\tau_{e_i}(\xi)}\right\}_{(\xi',\,\xi)\in ET\times ET}=0.$$

This assumption also makes sense. It means that various dividend tax rates τ_d are unaffected by various endowment tax rates τ_{e_i}.

The analysis of how a change in the stochastic endowment tax τ_{e_i} of an agent $i\in I$ will affect equilibrium asset prices $\overline{q}\left(\tau\right)$ is much easier than the effects of τ_d on $\overline{q}\left(\tau\right)$, since, under reasonable assumptions, increases in τ_{e_i} affect $\overline{q}\left(\tau\right)$ only through stochastic discount factors $\overline{\pi}(\tau)\in\mathbb{R}^{|ET|}_{++}$.

Let us first analyze how a change in the current $\tau_{e_i}\left(\xi\right)$ endowment tax rate of an agent $i\in I$ will affect current equilibrium asset prices $\overline{q}\left(\xi,\ \tau\right)$. We will start with the following lemma:

LEMMA 2.3.1: *Suppose assumptions of the Lemma 2.2.1. hold. Let*

$$\left(\left\{\left(\overline{c}_i,\ \overline{z}_i\right)\right\}_{i\in I},\ \left(\overline{p},\ \overline{q}\right)\right)\in\left(\mathbb{R}^{|ET\times L\times I|}_{+}\times\mathcal{Z}^{|I|}\right)\times\left(\mathbb{R}^{|ET\times L|}_{+}\times Q\right)$$

be a regular FM equilibrium in which markets are complete for an FM economy $\mathcal{E}(ET,\ (1-\overline{\tau}_e)\cdot e(\overline{\tau}_e),$ $\succeq,\ \delta,\ \mathcal{A}(\overline{\tau}_d))$. Let ξ be the initial node of the event tree ET. Then an open neighborhood $O_{\overline{\tau}}$ of $\overline{\tau}$ and a function

$$H(\tau)=\left(\left\{\left(\overline{c}_i(\tau),\ \overline{z}_i(\tau)\right)\right\}_{i\in I},\ \left(\overline{p}(\tau),\ \overline{q}(\tau)\right),\overline{\pi}(\tau)\right)\ \forall\tau\in O_{\overline{\tau}},$$

obtained in the Lemma 2.2.1. are s.t.

$$\boxed{\frac{\partial\overline{\pi}(\xi',\tau)}{\partial\tau_{e_i}\left(\overline{\xi}\right)}=\overline{\pi}(\xi',\tau)\cdot\left[rr_i(\overline{c}_i(\xi,\ 1,\ \tau))\cdot g_{\overline{c}_i(\xi,\,1,\,\tau)}-rr_i(\overline{c}_i(\xi',\ 1,\ \tau))\cdot g_{\overline{c}_i(\xi',\,1,\,\tau)}\right]\ \forall\overline{\xi}\in ET,}\qquad(5)$$

where

$$\frac{\partial\overline{\pi}}{\partial\tau_{e_i}}=\left\{\left\{\frac{\partial\overline{\pi}(\xi',\,\tau)}{\partial\tau_{e_i}(\xi,\,l)}\right\}_{l\in L}\right\}_{(\xi',\,\xi)\in ET\times ET}=\left\{\frac{\partial\overline{\pi}(\xi',\,\tau)}{\partial\tau_{e_i}(\xi)}\right\}_{(\xi',\,\xi)\in ET\times ET},$$

$$rr_i(c)=-\left[\frac{u_i''(c)\cdot c}{u_i'(c)}\right]$$

is the coefficient of relative risk aversion of an agent $i\in I$ and

$$g_{\overline{c}_i(\xi',\,1,\,\tau)}=\frac{1}{\overline{c}_i(\xi',\,1,\,\tau)}\cdot\frac{\partial\overline{c}_i(\xi',\,1,\,\tau)}{\partial\tau_{e_i}\left(\overline{\xi}\right)}\ \forall\xi'\in ET.$$

PROOF: It is similar to the proof of Lemma 2.2.2.

THEOREM 2.3.2: *Let*

$$\left(\{(\overline{c}_i,\ \overline{z}_i)\}_{i\in I},\ (\overline{p},\ \overline{q})\right) \in \left(\mathbb{R}_+^{|ET\times L\times I|} \times \mathcal{Z}^{|I|}\right) \times \left(\mathbb{R}_+^{|ET\times L|} \times Q\ \right)$$

be a regular FM equilibrium in which markets are complete for an FM economy $\mathcal{E}(ET,\ (1-\overline{\tau}_e)\cdot e(\overline{\tau}_e),\ \succeq,\ \delta,\ \mathcal{A}(\overline{\tau}_d))$. *Let* ξ *be the initial node of the event tree* ET. *Then an open neighborhood* $O_{\overline{\tau}}$ *of* $\overline{\tau}$ *and a function*

$$H(\tau) = \left(\{(\overline{c}_i(\tau),\ \overline{z}_i(\tau))\}_{i\in I},\ \ (\overline{p}(\tau),\ \overline{q}(\tau))\,,\ \overline{\pi}(\tau)\right)\ \forall \tau\in O_{\overline{\tau}},$$

obtained in the Lemma 2.2.1. are s.t.

$$\boxed{\frac{\partial \overline{q}(\xi,\,\tau)}{\partial \tau_{e_i}(\xi)} = \overline{q}\,(\xi,\ \tau)\cdot rr_i(\overline{c}_i(\xi,\,1,\,\tau))\cdot g_{\overline{c}_i(\xi,\,1,\,\tau)}}$$

and

$$\boxed{sign\ \boxed{\frac{\partial \overline{q}(\xi,\,\tau)}{\partial \tau_{e_i}(\xi)}} = sign\ \boxed{\frac{\partial \overline{c}_i(\xi,\,1,\,\tau)}{\partial \tau_{e_i}(\xi)}}\ \forall\,(\xi,\ i)\in ET\times I,}$$

where

$$\frac{\partial \overline{q}}{\partial \tau_{e_i}} = \left\{\left\{\frac{\partial \overline{q}(\xi,\,k,\,\tau)}{\partial \tau_{e_i}(\xi',\,l)}\right\}_{(k,\,l)\in K\times L}\right\}_{(\xi',\,\xi)\in ET\times ET} = \left\{\frac{\partial \overline{q}(\xi)}{\partial \tau_{e_i}(\xi')}\right\}_{(\xi',\,\xi)\in ET\times ET},$$

$$g_{\overline{c}_i(\xi,\,1,\,\tau)} = \frac{1}{\overline{c}_i(\xi,\,1,\,\tau)}\cdot\frac{\partial \overline{c}_i(\xi,\,1,\,\tau)}{\partial \tau_{e_i}(\xi)}.$$

PROOF: It is similar to the proof of Theorem 2.2.3.

The economic interpretation of the above result is as follows. Note first that an increase in $\tau_{e_i}(\xi,\ l)$ affects $\overline{q}\,(\xi,\ \tau)$ only through stochastic discount factors $\overline{\pi}(\xi',\ \tau)$, $\xi'\in ET^+(\xi)$. Although the sign of a derivative of current equilibrium asset prices $\overline{q}\,(\xi,\ \tau)$ with respect to current endowment tax rates $\tau_{e_i}(\xi)$ may be ambiguous, it is always the same as that of the derivative of the current equilibrium consumption $\overline{c}_i\,(\xi,\ 1,\ \tau)$ of the numeraire good 1 with respect to current endowment tax rates $\tau_{e_i}(\xi)$. Depending on the sign of $\frac{\partial \overline{c}_i(\xi,\,1,\,\tau)}{\partial \tau_{e_i}(\xi)}$, an increase in $\tau_{e_i}(\xi)$ might be reducing or boosting $\overline{q}\,(\xi,\ \tau)$. Therefore, we have two cases to consider here:

Suppose first $\frac{\partial \overline{c}_i(\xi,\,1,\,\tau)}{\partial \tau_{e_i}(\xi)} \leqslant 0$, i.e., the numeraire good 1 is a normal good. Then an increase in $\tau_{e_i}(\xi)$ reduces $\overline{\pi}(\xi',\ \tau)$, thus decreasing today's price of future consumption $\overline{c}_i\,(\xi',\ 1,\ \tau)$, $\xi'\in ET^+(\xi)$ of the numeraire good 1. Since financial assets represent claims on future consumption, the increase in $\tau_{e_i}(\xi)$ reduces today's asset prices $\overline{q}\,(\xi,\ \tau)$.

Suppose now $\frac{\partial \overline{c}_i(\xi,\,1,\,\tau)}{\partial \tau_{e_i}(\xi)} > 0$, i.e., the numeraire good 1 is an inferior good. Then an increase in $\tau_{e_i}(\xi)$ boosts $\overline{\pi}(\xi',\ \tau)$, thus increasing today's price of future consumption $\overline{c}_i\,(\xi',\ 1,\ \tau)$, $\xi'\in ET^+(\xi)$ of the numeraire good 1. Since financial assets represent claims on future consumption, the increase in $\tau_{e_i}(\xi)$ boosts today's asset prices $\overline{q}\,(\xi,\ \tau)$.

While inferior goods have been demonstrated to exist, it is generally believed that they are rare: at any given time and price level, the demand for the vast majority of goods moves in the intuitive direction with respect to changes in after-tax income. So it is reasonable to assume that the numeraire good 1 is a normal good. Therefore, it is natural to conclude that under reasonable assumptions, an increase in the current endowment tax rate reduces current asset prices.

COROLLARY 2.3.3: *Suppose assumptions of the above Theorem 2.3.2. hold. Assume further that all agents are identical and exhibit CRRA, i.e.,*

$$rr_i(c) = -\left[\frac{u''(c)\cdot c}{u'(c)}\right] = a\ \forall i\in I.$$

Agents' endowments are taxed identically, i.e.,

$$\tau_{e_i}(\xi) = \tau_{e_{\bar{i}}}(\xi)\ \forall\,(\xi,\ i) \in ET \times I.$$

In addition, assets pay zero dividends, i.e.,

$$d(\xi,\ k,\ \tau_d) = 0\ \forall\,(\xi,\ k) \in ET \times K^{12}$$

and

$$\frac{\partial e}{\partial \tau_e} = 0.$$

Then

$$\boxed{E_{q(\xi,\ \tau),\ 1-\tau_e(\xi)} = a,}$$

where

$$E_{q(\xi,\ \tau),\ 1-\tau_{e_{\bar{i}}}(\xi)} = \frac{\frac{1}{\overline{q}(\xi,\ \tau)}\frac{\partial \overline{q}(\xi,\ \tau)}{\partial \tau_{e_{\bar{i}}}(\xi)}}{\frac{1}{(1-\tau_{e_{\bar{i}}}(\xi))}\frac{\partial (1-\tau_{e_{\bar{i}}}(\xi))}{\partial \tau_{e_{\bar{i}}}(\xi)}}\ \forall \xi \in ET$$

is the elasticity of asset prices $q(\xi,\tau)$ with respect to the economic freedom $1-\tau_{e_{\bar{i}}}(\xi)$ at a node $\xi \in ET$.

PROOF: It is similar to the proof of Corollary 2.2.4.

Let us now analyze how a change in a future $\tau_{e_i}\left(\overline{\xi}\right)$, $\overline{\xi} \in ET^+(\xi)$ stochastic endowment tax rate of an agent $i \in I$ will affect current equilibrium asset prices $\overline{q}\,(\xi,\ \tau)$. Since a change in $\tau_{e_i}\left(\overline{\xi}\right)$ might affect various node prices $\overline{\pi}(\xi',\ \tau)$, $\xi' \in ET^+(\xi)$ differently, the net effect of $\tau_{e_i}\left(\overline{\xi}\right)$ on $\overline{q}\,(\xi,\ \tau)$ is ambiguous. Unlike the comparative statics of $\overline{q}\,(\xi,\ \tau)$ with respect to $\tau_{e_i}(\xi)$, however, it does not appear to be possible to derive economically intuitive comparative statics of $\overline{q}\,(\xi,\ \tau)$ with respect to $\tau_{e_i}\left(\overline{\xi}\right)$ results without either assuming CRRA utility functions or identical agents.

Suppose now that agents exhibit CRRA but are not necessarily identical. Our goal here is to determine the sign of

$$\frac{\partial \overline{q}(\xi,\ \tau)}{\partial \tau_{e_i}\left(\overline{\xi},\ 1\right)} = \left\{\frac{\partial \overline{q}(\xi,\ k,\ \tau)}{\partial \tau_{e_i}\left(\overline{\xi},\ 1\right)}\right\}_{k \in K} \in \mathbb{R}^{|K|}\ \forall \overline{\xi} \in ET^+(\xi).$$

THEOREM 2.3.4: *Let*

$$\left(\left\{(\overline{c}_i,\ \overline{z}_i)\right\}_{i\in I},\ (\overline{p},\ \overline{q})\right) \in \left(\mathbb{R}_+^{|ET\times L\times I|} \times \mathcal{Z}^{|I|}\right) \times \left(\mathbb{R}_+^{|ET\times L|} \times Q\right)$$

be a regular FM equilibrium in which markets are complete for an FM economy $\mathcal{E}(ET,\ (1-\overline{\tau}_e)\cdot e(\overline{\tau}_e),\ \succeq,\ \delta,\ \mathcal{A}(\overline{\tau}_d))$, s.t. $\frac{\partial e}{\partial \tau_e} = 0$ and agents' preferences $\succeq_i$ on $\mathbb{R}_+^{|ET\times L|} \times \mathbb{R}_+^{|ET\times L|}$ are given by the utility function

$$U_i(c_i,\ G) = \sum_{(\xi,\ l)\in ET\times L} \Pr(\xi)\cdot b_i^{T(\xi)} \cdot \left[u_i(c_i(\xi,\ l)) + v_i(G(\xi,\ l))\right]\ \forall i \in I,$$

where u_i is a CRRA utility function such that $u_i(c) = \frac{c^{1-a_i}}{1-a_i}$.
Assume further that an open neighborhood $O_{\overline{\tau}}$ of $\overline{\tau}$ and a function

$$H(\tau) = \left(\left\{(\overline{c}_i(\tau),\ \overline{z}_i(\tau))\right\}_{i\in I},\ \left(\overline{p}(\tau),\ \overline{q}(\tau)\right),\overline{\pi}(\tau)\right)\ \forall \tau \in O_{\overline{\tau}},$$

[12] We can obtain a similar result without assuming that assets pay zero dividends. It is sufficient to assume $\tau_d(\xi) = \tau_e(\xi)$ $\forall \xi \in ET$ instead.

obtained in the Lemma 2.2.1. are s.t.

$$\frac{\overline{c}_i(\xi',\, 1,\, \tau)}{\overline{c}_i(\xi,\, 1,\, \tau)} = \frac{\sum\limits_{i\in I}\big(1-\tau_{e_i}(\xi',\, 1)\big)\cdot e_i(\xi',\, 1,\, \tau_{e_i})+\sum\limits_{k\in K}\delta(k)\cdot(1-\tau_d(\xi',\, k))\cdot d(\xi',\, k,\, \tau_d)}{\sum\limits_{i\in I}\big(1-\tau_{e_i}(\xi,\, 1)\big)\cdot e_i(\xi,\, 1,\, \tau_{e_i})+\sum\limits_{k\in K}\delta(k)\cdot(1-\tau_d(\xi,\, k))\cdot d(\xi,\, k,\, \tau_d)}$$

$\forall\,(\xi,\ \xi',\ \tau,\ i) \in ET \times ET \times O_{\overline{\tau}} \times I$, *where* $\delta(k)$ *is the total number of outstanding shares of asset* $k \in K$*. Let* ξ *be the initial node of the event tree* ET*. Fix* $\overline{\xi} \in ET^+(\xi)$*. Then*

$$\boxed{\frac{\partial \overline{q}(\xi,\, k,\, \tau)}{\partial \tau_{e_i}\big(\overline{\xi},\, 1\big)} = a_i \cdot \overline{\pi}(\overline{\xi},\, \tau) \cdot \Pi(\overline{\xi},\, i,\, \tau) \cdot \big(1 - \tau_d\left(\overline{\xi},\ k\right)\big) \cdot d\left(\overline{\xi},\ \tau_d\right) > 0\ \forall \overline{\xi} \in ET^+(\xi),}$$

where

$$\Pi(\overline{\xi},\, i,\, \tau) = \frac{e_i(\overline{\xi},\, 1,\, \tau_{e_i})}{\sum\limits_{i\in I}\big(1-\tau_{e_i}(\overline{\xi},\, 1)\big)\cdot e_i(\overline{\xi},\, 1,\, \tau_{e_i})+\sum\limits_{k\in K}\delta(k)\cdot\big(1-\tau_d(\overline{\xi},\, k)\big)\cdot d(\overline{\xi},\, k,\, \tau_d)}.$$

PROOF: See Appendix B.

The economic interpretation of the above result is as follows. Fix $\overline{\xi} \in ET^+(\xi)$. An increase in $\tau_{e_i}\left(\overline{\xi},\ 1\right)$ boosts stochastic discount factors $\overline{\pi}(\overline{\xi},\ \tau)$, $\xi' \in ET^+(\xi)$ without affecting after-tax dividends $\left(1 - \tau_d\left(\overline{\xi},\ k\right)\right) \cdot d\left(\overline{\xi},\ k,\ \tau_d\right)$, thus boosting asset prices $\overline{q}\,(\xi,\ k,\ \tau)$.

Suppose now agents are identical but do not necessarily exhibit CRRA.

THEOREM 2.3.5.: *Suppose assumptions of the above Theorem 2.3.4. hold, except that now identical agents' preferences* $\succeq_i$ *on* $\mathbb{R}_+^{|ET\times L|} \times \mathbb{R}_+^{|ET\times L|}$ *are given by the utility function*

$$U_i(c,\, G) = \sum_{(\xi,\, l)\in ET\times L} \Pr(\xi) \cdot b^{T(\xi)} \cdot [u(c(\xi,\ l)) + v(G(\xi,\ l))]\ \forall i \in I,$$

where $u \in C^2$ *such that* $u'(\cdot) > 0$ *and* $u''(\cdot) < 0$*.*

Let ξ *be the initial node of the event tree* ET*. Fix* $\overline{\xi} \in ET^+(\xi)$*. Then*

$$\boxed{\frac{\partial \overline{q}(\xi,\, \tau)}{\partial \tau_e\big(\overline{\xi}\big)} = \overline{\pi}(\overline{\xi}, \tau) \cdot \left[rr(\overline{c}(\overline{\xi},\ 1,\ \tau))\right] \cdot \frac{e(\overline{\xi},\, 1,\, \tau_e)}{|I|\cdot\overline{c}(\overline{\xi},\, 1,\, \tau)} \cdot \left(1 - \tau_d\left(\overline{\xi}\right)\right) \cdot d\left(\overline{\xi},\ \tau_d\right) > 0.}$$

PROOF: It is similar to the proof of Theorem 2.3.4.

The economic interpretation of this result is the same as for Theorem 2.3.4.

3. CONCLUSION

This paper studies comparative statics of FM equilibria in the finite horizon GEI model with respect to changes in stochastic tax rates imposed on agents' endowments and dividends. We show that under reasonable assumptions, without assuming CRRA and identical agents, an increase in the current dividend tax rate unambiguously reduces current asset prices. The paper also finds that there exists a bound $\overline{B}$ such that for a coefficient of relative risk aversion less than $\overline{B}$, an increase in a future dividend tax rate reduces current price of tradable assets. At the same time, for a coefficient of relative risk aversion greater than $\overline{B}$, an increase in a future dividend tax rate boosts current price of tradable assets. Finally, for a coefficient of relative risk aversion equal to $\overline{B}$, an increase in a future dividend tax rate leaves current price of tradable assets unchanged. As a special case, under additional assumptions, $\overline{B}$ is equal to 1. Also, under reasonable assumptions, an increase in the current endowment tax rate reduces current asset prices, while an increase in a future endowment tax rate boosts current asset prices.

4. APPENDIX A

Additional Definitions

We need to introduce several additional notions to define finite horizon FM Economies with stochastic taxation $\tau = (\tau_e,\ \tau_d)$ imposed on agents' endowments and assets' dividends and used to finance spending on the public good G.

Suppose there is an event-tree ET and a set I of finitely living investors-consumers who trade a set L of commodities on spot markets and a set K of assets on financial markets, such that

$$max\left[|ET|,\ \ |I|,\ \ |L|,\ \ |K|\right] < \infty.$$

First, we discuss the asset structure of our model.

DEFINITION: *Let $\xi(k) \in ET$ be the node of issue for an asset $k \in K$. Define the set ζ of all nodes of issue of existing financial contracts as*

$$\zeta = \{\xi(k) \mid k \in K\}.$$

DEFINITION: *Let*

$$d\left(\xi,\ k,\ \tau_d\right) \in \mathbb{R}$$

be the real before-tax dividend of asset $k \in K$ paid in units of the numeraire good 1 at node $\xi \in ET$, where

$$d(\xi,\, k,\, \tau_d) = 0\ \ \forall \xi \in ET \setminus ET^{+}(\xi(k))\ \forall k \in K,$$

i.e., an asset $k \in K$ issued at node $\xi(k) \in ET$ pays no dividends prior to or at node $\xi(k) \in ET$,

$$d(\tau_d) = \left\{d\left(\xi,\ \tau_d\right)\right\}_{\xi \in ET} \in \mathbb{R}^{|ET \times K|}$$

be the matrix of real before-tax dividends of the $|K|$ assets paid in units of the numeraire good 1.

DEFINITION: *Let ζ be the set of all nodes of issue of $|K|$ existing financial contracts and d be the $|ET \times K|$ matrix of dividends. Then we call the pair*

$$\mathcal{A}(\tau_d) = (\zeta,\, (1-\tau_d)\cdot d(\tau_d))$$

the financial structure.

We are now ready to define a finite horizon FM Economy with stochastic taxation $\tau = (\tau_e,\, \tau_d) \in [0,\ 1]^{|ET \times L \times I|} \times [0,\ 1]^{|ET \times K|}$ and with assets in strictly positive supply $\delta = \{\delta(k)\}_{k \in K} \in \mathbb{R}^{|K|}_{++}$.

DEFINITION: *We denote by $\mathcal{E}(ET,\, (1-\tau_e)\cdot e\left(\tau_e\right),\, \succeq,\, \delta,\, \mathcal{A}(\tau_d))$ with*

$$\mathcal{A}(\tau_d) = (\zeta,\, (1-\tau_d)\cdot d(\tau_d))$$

a finite horizon FM Economy with stochastic taxation $\tau = (\tau_e,\, \tau_d) \in [0,\ 1]^{|ET \times L \times I|} \times [0,\ 1]^{|ET \times K|}$ and with assets in strictly positive supply $\delta = \{\delta(k)\}_{k \in K} \in \mathbb{R}^{|K|}_{++}$, where agents' preferences $\succeq_i$ are given by the utility function

$$U_i : \mathbb{R}^{|ET \times L|}_{+} \times \mathbb{R}^{|ET \times L|}_{+} \longrightarrow \mathbb{R}$$

such that

$$U_i(c_i, G) = \sum_{(\xi,\, l) \in ET \times L} \Pr(\xi) \cdot b_i^{T(\xi)} \cdot [u_i(c_i(\xi,\ l)) + v_i(G(\xi,\ l))]\ \forall i \in I,$$

where the spending on the public good $G = \{G(\xi,\ l)\}_{(\xi,\, l) \in ET \times L} \in \mathbb{R}_+^{|\mathbb{ET} \times \mathbb{L}|}$ *is given by*

$$G(\xi,\ l) =$$
$$\begin{cases} \sum_{i \in I} \tau_{e_i}(\xi,\ l) \cdot e_i(\xi,\ l,\ \tau_{e_i}) & \forall (\xi,\ l) \in ET \times [L \setminus \{1\}] \\ \left[\sum_{i \in I} \tau_{e_i}(\xi,\ l) \cdot e_i(\xi,\ l,\ \tau_{e_i}) + \sum_{k \in K} \tau_d(\xi,\ k) \cdot d(\xi,\ k,\ \tau_d) \cdot \delta(k) \right] & \forall (\xi,\ l) \in ET \times \{1\} \end{cases}$$

and

$$\sum_{i \in I} z_i = \delta = \{\delta(k)\}_{k \in K} \in \mathbb{R}_{++}^{|K|}.$$

5. APPENDIX B

Proofs for Comparative Statics of FM Equilibria with Respect to the Dividend Tax τ_d

PROOF OF LEMMA 2.2.1: We know that the total excess demand function

$$ED: \underbrace{\left[\mathbb{R}_{++}^{|ET \times L|} \times Q\right] \times \left[\mathbb{R}_{++}^{|ET \times L \times I|} \times \mathbb{R}^{|ET \times K|}\right]}_{Open\ Subset\ of\ \left[\mathbb{R}^{|ET \times L|} \times \mathbb{R}^{|ET \times K|}\right] \times \left[\mathbb{R}^{|ET \times L \times I|} \times \mathbb{R}^{|ET \times K|}\right]} \longrightarrow \left[\mathbb{R}^{|ET \times L|} \times \mathbb{R}^{|ET \times K|}\right]$$

given by

$$ED(p,\ q,\ (1 - \tau_e) \cdot e(\tau_e),\ (1 - \tau_d) \cdot d(\tau_d)) =$$
$$\left[\underbrace{ED_C(p,\ q,\ (1 - \tau_e) \cdot e(\tau_e),\ (1 - \tau_d) \cdot d(\tau_d))}_{Excess\ Demand\ for\ Commodities} \right] \times \left[\underbrace{ED_A(p,\ q,\ (1 - \tau_e) \cdot e(\tau_e),\ (1 - \tau_d) \cdot d(\tau_d))}_{Excess\ Demand\ for\ Assets} \right].$$

is such that

$$ED \in C^\infty(\left[\mathbb{R}_{++}^{|ET \times L|} \times Q\right] \times \left[\mathbb{R}_{++}^{|ET \times L \times I|} \times \mathbb{R}^{|ET \times K|}\right])^{13}.$$

Let

$$(\{(\overline{c}_i,\ \overline{z}_i)\}_{i \in I},\ (\overline{p},\ \overline{q})) \in \left(\mathbb{R}_+^{|ET \times L \times I|} \times \mathcal{Z}^{|I|}\right) \times \left(\mathbb{R}_{++}^{|ET \times L|} \times Q\ \right)$$

be a regular FM equilibrium in which markets are complete for an FM economy $\mathcal{E}(ET,\ (1 - \overline{\tau}_e) \cdot e(\overline{\tau}_e),\ \succeq,\ \delta,\ \mathcal{A}(\overline{\tau}_d))$. Therefore,

$$ED(\overline{p},\ \overline{q},\ (1 - \overline{\tau}_e) \cdot e(\overline{\tau}_e),\ (1 - \overline{\tau}_d) \cdot d(\overline{\tau}_d)) = 0$$

and

$$det \left[D_{(p,\ q)} ED(\overline{p},\ \overline{q},\ (1 - \overline{\tau}_e) \cdot e(\overline{\tau}_e),\ (1 - \overline{\tau}_d) \cdot d(\overline{\tau}_d))\right] \neq 0.$$

The Classical Implicit Function Theorem (IFT) states:

Let $X \times A$ *be an open subset* of $\mathbb{R}^n \times \mathbb{R}^m$, *and function* $f : X \times A \longrightarrow \mathbb{R}^n$ *be s.t.* $f \in C^\infty(X \times A)$. *Let* $\overline{y} = f(\overline{x}, \overline{a})$ *and* $\exists D_x^{-1} f(\overline{x}, \overline{a})$. *Then* $\exists$ *open neighborhoods* $U_{\overline{x}}$ *of* $\overline{x}$ *and* $W_{\overline{a}}$ *of* $\overline{a}$ *s.t.* $\exists$ *a !function*

[13] See Magin (2017)

$$
\begin{gathered}
g : W_{\overline{a}} \longrightarrow U_{\overline{x}},\ s.t. \\
g(\overline{a}) = \overline{x}, \\
f(g(a),\, a) = \overline{y}\ \forall a \in W_{\overline{a}}, \\
g \in C^{\infty}\,(W_{\overline{a}}), \\
\tfrac{\partial g}{\partial a} = \left[\tfrac{\partial f}{\partial x}\right]^{-1} \tfrac{\partial f}{\partial a}.
\end{gathered}
$$

Therefore, we can conclude by the IFT that $\exists$ open neighborhoods U of $(\overline{p},\ \overline{q}) \in \mathbb{R}_{++}^{|ET\times L|} \times Q$ and W of $((1-\overline{\tau}_e)\cdot e(\overline{\tau}_e),\ (1-\overline{\tau}_d)\cdot d(\overline{\tau}_d)) \in \mathbb{R}_{++}^{|ET\times L\times I|} \times \mathbb{R}^{|ET\times K|}$ and a function

$$
\begin{gathered}
g : W \longrightarrow U, \text{ s.t.} \\
g(\,(1-\overline{\tau}_e)\cdot e(\overline{\tau}_e),\ (1-\overline{\tau}_d)\cdot d(\overline{\tau}_d)) = (\overline{p},\ \overline{q}), \\
ED(g(\,(1-\tau_e)\cdot e(\tau_e),\ (1-\tau_d)\cdot d(\tau_d)),\ (1-\tau_e)\cdot e(\tau_e),\ (1-\tau_d)\cdot d(\tau_d)) = 0
\end{gathered}
$$

$\forall\,((1-\tau_e)\cdot e(\tau_e),\ (1-\tau_d)\cdot d(\tau_d)) \in W$ and

$$g \in C^{\infty}\,(W).$$

We know that the function

$$f : [0,\ 1]^{|ET\times L\times I|} \times [0,\ 1]^{|ET\times K|} \longrightarrow \mathbb{R}^{|ET\times L\times I|} \times \mathbb{R}^{|ET\times K|}$$

defined as

$$f(\tau_e,\, \tau_d) = ((1-\tau_e)\cdot e(\tau_e),\ (1-\tau_d)\cdot d(\tau_d))$$

is such that

$$f \in C^1\left([0,\ 1]^{|ET\times L\times I|} \times [0,\ 1]^{|ET\times K|}\right).$$

Therefore, since $W \subset \mathbb{R}_{++}^{|ET\times L\times I|} \times \mathbb{R}^{|ET\times K|}$ is an open set, the set

$$O_{\overline{\tau}} = f^{-1}(W)$$

is an open set as well. Since

$$f : [0,\ 1]^{|ET\times L\times I|} \times [0,\ 1]^{|ET\times K|} \longrightarrow f([0,\ 1]^{|ET\times L\times I|} \times [0,\ 1]^{|ET\times K|})$$

is onto, we can conclude that

$$f(O_{\overline{\tau}}) = f\left(f^{-1}(W)\right) = W.$$

Thus,

$$g(f(O_{\overline{\tau}})) = g\,(W).$$

Hence, the function

$$[g \circ f] : O_{\overline{\tau}} \longrightarrow g\,(W)$$

defined as

$$[g \circ f]\,(\tau) = g\,(f(\tau)) = (\overline{p}(\tau),\ \overline{q}(\tau))\ \forall \tau \in O_{\overline{\tau}}$$

is onto and

$$[g \circ f] \in C^1\left(O_{\overline{\tau}}\right).$$

We know from Magin (2017) that the individual demand function for consumption

$$\overline{c}_i : \left[\mathbb{R}_{++}^{|ET \times L|} \times Q\right] \times \left[\mathbb{R}_{++}^{|ET \times L|} \times \mathbb{R}^{|ET \times K|}\right] \longrightarrow \mathbb{R}_{+}^{|ET \times L|}$$

given by

$$\begin{gathered} \overline{c}_i(p,\ q,\ (1-\tau_{e_i}) \cdot e(\tau_{e_i}),\ (1-\tau_d) \cdot d(\tau_d)) = \\ \arg\max\left\{U_i(c_i) \mid c_i \in B(p,\ q,\ \ (1-\tau_{e_i}) \cdot e(\tau_{e_i}),\ (1-\tau_d) \cdot d(\tau_d))\right\} \end{gathered}$$

and the individual demand function for assets

$$\overline{z}_i : \left[\mathbb{R}_{++}^{|ET \times L|} \times Q\right] \times \left[\mathbb{R}_{++}^{|ET \times L|} \times \mathbb{R}^{|ET \times K|}\right] \longrightarrow \mathbb{R}^{|ET \times L|}$$

given by

$$\begin{gathered} \overline{z}_i(p,\ q,\ (1-\tau_{e_i}) \cdot e(\tau_{e_i}),\ (1-\tau_d) \cdot d(\tau_d)) = \\ \{z_i \in \mathcal{Z} \mid p \cdot (\overline{c}_i(p,\ q,\ \ (1-\tau_{e_i}) \cdot e(\tau_{e_i}),\ (1-\tau_d) \cdot d(\tau_d)) - (1-\tau_{e_i}) \cdot e(\tau_{e_i})) = W(q,\ \ (1-\tau_d) \cdot d(\tau_d)) \cdot z_i\} \end{gathered}$$

are such that

$$\overline{c}_i,\, \overline{z}_i \in C^\infty(\left[\mathbb{R}_{++}^{|ET \times L|} \times Q\right] \times \left[\mathbb{R}_{++}^{|ET \times L|} \times \mathbb{R}^{|ET \times K|}\right]\ )\ \forall i \in I.$$

Hence, the individual equilibrium consumption of agent $i \in I$

$$\overline{c}_i : O_{\overline{\tau}} \longrightarrow \overline{c}_i(O_{\overline{\tau}})$$

defined with some abuse of notations as

$$\overline{c}_i(\tau) = \overline{c}_i(\overline{p}(\tau),\, \overline{q}(\tau),\, (1-\tau_e) \cdot e(\tau_e),\ (1-\tau_d) \cdot d(\tau_d))\ \forall \tau \in O_{\overline{\tau}}$$

and the individual equilibrium portfolio of agent $i \in I$

$$\overline{z}_i : O_{\overline{\tau}} \longrightarrow \overline{c}_i(O_{\overline{\tau}})$$

defined with some abuse of notations as

$$\overline{z}_i(\tau) = \overline{z}_i(\overline{p}(\tau),\, \overline{q}(\tau),\, (1-\tau_e) \cdot e(\tau_e),\ (1-\tau_d) \cdot d(\tau_d))\ \forall \tau \in O_{\overline{\tau}}$$

are such that

$$\overline{c}_i,\, \overline{z}_i \in C^1\left(O_{\overline{\tau}}\right)\ \forall i \in I.$$

Define

$$h : O_{\overline{\tau}} \longrightarrow \left(\mathbb{R}_{+}^{|ET \times L \times I|} \times \mathcal{Z}^{|I|}\right) \times \left(\mathbb{R}_{+}^{|ET \times L|} \times Q\ \right)$$

as

$$h(\tau) = (\{(\overline{c}_i(\tau),\ \overline{z}_i(\tau))\}_{i \in I}\,, (\overline{p}(\tau),\ \overline{q}(\tau)))\ \forall \tau \in O_{\overline{\tau}},$$

where

$$\left(\{(\overline{c}_i(\tau),\ \overline{z}_i(\tau))\}_{i\in I},(\overline{p}(\tau),\ \overline{q}(\tau))\right) \in \left(\mathbb{R}_+^{|ET\times L\times I|}\times \mathcal{Z}^{|I|}\right)\times\left(\mathbb{R}_+^{|ET\times L|}\times Q\ \right)$$

be an FM equilibrium for an FM economy $\mathcal{E}(ET,\ (1-\tau_e)\cdot e(\tau_e),\ \succeq,\ \delta,\ \mathcal{A}(\tau_d))$. Moreover, as Magin (2015b) demonstrated, we can always make $O_{\overline{\tau}}$ sufficiently small so that this FM equilibrium is complete $\forall \tau\in O_{\overline{\tau}}$. Clearly, $h\in C^1\left(O_{\overline{\tau}}\right)$. ■

PROOF OF LEMMA 2.2.2: Let us set up Lagrangian

$$\pounds_i = U_i(c_i,\,G) - \lambda_i\cdot[p\cdot c_i - p\cdot(1-\tau_{e_i})\cdot e(\tau_{e_i}) - W(q,\,(1-\tau_d)\cdot d(\tau_d))\cdot z_i]\ \forall i\in I,$$

where $\lambda_i\in\mathbb{R}^{|ET|}$ is the Lagrangian multiplier. Taking first-order conditions we obtain

$$D\pounds_i = \begin{cases} \frac{\partial \pounds_i}{\partial c_i} = DU_i(c_i) - \lambda_i\cdot p = 0 \\ \frac{\partial \pounds_i}{\partial z_i} = \lambda_i\cdot W(q,\,(1-\tau_d)\cdot d(\tau_d)) = 0 \\ \frac{\partial \pounds_i}{\partial \lambda_i} = -p\cdot c_i + p\cdot(1-\tau_{e_i})\cdot e(\tau_{e_i}) + W(q,\,(1-\tau_d)\cdot d(\tau_d))\cdot z_i = 0 \end{cases}$$

Therefore, substituting equilibrium consumption of agent $i\in I$ back into the first-order conditions and taking into consideration that markets are complete we obtain

$$\boxed{\overline{\pi}(\xi',\,\tau) = \overline{\pi}_i(\xi',\,\tau) = \frac{\lambda_i(\xi')}{\lambda_i(\xi)} = b_i^{T(\xi')}\cdot\frac{u_i'(\overline{c}_i(\xi',\,1,\,\tau))}{u_i'(\overline{c}_i(\xi,\,1,\,\tau))}\cdot \Pr(\xi')\ \forall(\xi',\,i)\in ET^+(\xi)\times I.} \quad (1)$$

and

$$\boxed{\overline{q}\,(\xi,\ \tau) = \sum_{\xi'\in ET^+(\xi)} \underbrace{b_i^{T(\xi')}\cdot\frac{u_i'\left(\overline{c}_i(\xi',\ 1,\ \tau)\right)}{u_i'\left(\overline{c}_i(\xi,\ 1,\ \tau)\right)}\cdot \Pr(\xi')}_{\overline{\pi}(\xi',\,\tau)}\cdot(1-\tau_d\,(\xi'))\cdot d\,(\xi',\ \tau_d)\ \forall \xi\in ET.} \quad (2)$$

Differentiating $\overline{\pi}(\xi',\,\tau)$ with respect to $\tau_d\left(\overline{\xi}\right)$ we obtain

$$\frac{\partial\overline{\pi}(\xi',\,\tau)}{\partial\tau_d\left(\overline{\xi}\right)} = b_i^{T(\xi')}\cdot\Pr(\xi')\cdot$$
$$\cdot\left[\frac{u_i''(\overline{c}_i(\xi',\,1,\,\tau))\cdot\frac{\partial\overline{c}_i(\xi',\ 1,\ \tau)}{\partial\tau_d\left(\overline{\xi}\right)}\cdot u_i'(\overline{c}_i(\xi,\,1,\,\tau)) - u_i'(\overline{c}_i(\xi',\,1,\,\tau))\cdot u_i''(\overline{c}_i(\xi,\,1,\,\tau))\cdot\frac{\partial\overline{c}_i(\xi,\,1,\,\tau)}{\partial\tau_d\left(\overline{\xi}\right)}}{\left(u_i'(\overline{c}_i(\xi,\,1,\,\tau))\right)^2}\right] =$$
$$= b_i^{T(\xi')}\cdot\Pr(\xi')\cdot$$
$$\cdot\left[-\frac{u_i'(\overline{c}_i(\xi',\,1,\,\tau))\cdot u_i''(\overline{c}_i(\xi,\,1,\,\tau))}{\left(u_i'(\overline{c}_i(\xi,\,1,\,\tau))\right)^2}\frac{\partial\overline{c}_i(\xi,\,1,\,\tau)}{\partial\tau_d\left(\overline{\xi}\right)} + \frac{u_i''(\overline{c}_i(\xi',\,1,\,\tau))\cdot u_i'(\overline{c}_i(\xi,\,1,\,\tau))}{\left(u_i'(\overline{c}_i(\xi,\,1,\,\tau))\right)^2}\frac{\partial\overline{c}_i(\xi',\ 1,\ \tau)}{\partial\tau_d\left(\overline{\xi}\right)}\right] =$$
$$= b_i^{T(\xi')}\cdot\Pr(\xi')\cdot$$
$$\left[\frac{u_i'(\overline{c}_i(\xi',\,1,\,\tau))}{u_i'(\overline{c}_i(\xi,\,1,\,\tau))}\cdot[-\frac{u_i''(\overline{c}_i(\xi,\,1,\,\tau))}{\left(u_i'(\overline{c}_i(\xi,\,1,\,\tau))\right)}]\cdot\frac{\partial\overline{c}_i(\xi,\,1,\,\tau)}{\partial\tau_d\left(\overline{\xi}\right)} - \frac{u_i'(\overline{c}_i(\xi',\,1,\,\tau))}{u_i'(\overline{c}_i(\xi,\,1,\,\tau))}\cdot[-\frac{u_i''(\overline{c}_i(\xi',\,1,\,\tau))}{u_i'(\overline{c}_i(\xi',\,1,\,\tau))}]\cdot\frac{\partial\overline{c}_i(\xi',\ 1,\ \tau)}{\partial\tau_d\left(\overline{\xi}\right)}\right] =$$
$$= \left[b_i^{T(\xi')}\cdot\frac{u_i'(\overline{c}_i(\xi',\,1,\,\tau))}{u_i'(\overline{c}_i(\xi,\,1,\,\tau))}\cdot\Pr(\xi')\right]\cdot\left[[-\frac{u_i''(\overline{c}_i(\xi,\,1,\,\tau))}{\left(u_i'(\overline{c}_i(\xi,\,1,\,\tau))\right)}]\cdot\frac{\partial\overline{c}_i(\xi,\,1,\,\tau)}{\partial\tau_d\left(\overline{\xi}\right)} - [-\frac{u_i''(\overline{c}_i(\xi',\,1,\,\tau))}{u_i'(\overline{c}_i(\xi',\,1,\,\tau))}]\cdot\frac{\partial\overline{c}_i(\xi',\ 1,\ \tau)}{\partial\tau_d\left(\overline{\xi}\right)}\right] =$$
$$= \overline{\pi}(\xi',\,\tau)\cdot\left[[-\frac{u_i''(\overline{c}_i(\xi,\,1,\,\tau))}{\left(u_i'(\overline{c}_i(\xi,\,1,\,\tau))\right)}]\cdot\frac{\partial\overline{c}_i(\xi,\,1,\,\tau)}{\partial\tau_d\left(\overline{\xi}\right)} - [-\frac{u_i''(\overline{c}_i(\xi',\,1,\,\tau))}{u_i'(\overline{c}_i(\xi',\,1,\,\tau))}]\cdot\frac{\partial\overline{c}_i(\xi',\ 1,\ \tau)}{\partial\tau_d\left(\overline{\xi}\right)}\right] =$$

$$= \overline{\pi}(\xi', \tau) \cdot \left[\underbrace{[-\frac{u_i''(\overline{c}_i(\xi,\ 1,\ \tau))}{u_i'(\overline{c}_i(\xi,\ 1,\ \tau))} \cdot \overline{c}_i(\xi,\ 1,\ \tau)]}_{rr_i(\overline{c}_i(\xi,\ 1,\ \tau))} \cdot \underbrace{[\frac{1}{\overline{c}_i(\xi,\ 1,\ \tau)} \frac{\partial \overline{c}_i(\xi,\ 1,\ \tau)}{\partial \tau_d\left(\overline{\xi}\right)}]}_{g_{\overline{c}_i(\xi,\ 1,\ \tau)}} \right.$$

$$\left. - \underbrace{[-\frac{u_i''(\overline{c}_i(\xi',\ 1,\ \tau))}{u_i'(\overline{c}_i(\xi',\ 1,\ \tau))} \cdot \overline{c}_i(\xi',\ 1,\ \tau)]}_{rr_i(\overline{c}_i(\xi',\ 1,\ \tau))} \cdot \underbrace{[\frac{1}{\overline{c}_i(\xi',\ 1,\ \tau)} \frac{\partial \overline{c}_i(\xi',\ 1,\ \tau)}{\partial \tau_d\left(\overline{\xi}\right)}]}_{g_{\overline{c}_i(\xi',\ 1,\ \tau)}} \right].$$

So

$$\boxed{\frac{\partial \overline{\pi}(\xi',\ \tau)}{\partial \tau_d\left(\overline{\xi}\right)} = \overline{\pi}(\xi', \tau) \cdot \left[rr_i(\overline{c}_i(\xi,\ 1,\ \tau)) \cdot g_{\overline{c}_i(\xi,\ 1,\ \tau)} - rr_i(\overline{c}_i(\xi',\ 1,\ \tau)) \cdot g_{\overline{c}_i(\xi',\ 1,\ \tau)}\right] \forall \overline{\xi} \in ET,} \tag{3}$$

where

$$g_{\overline{c}_i(\xi',\ 1,\ \tau)} = \frac{1}{\overline{c}_i(\xi',\ 1,\ \tau)} \cdot \frac{\partial \overline{c}_i(\xi',\ 1,\ \tau)}{\partial \tau_d\left(\overline{\xi}\right)}\ \forall \xi' \in ET.\ \blacksquare$$

PROOF OF THEOREM 2.2.3: Note first that by Assumption D3

$$\frac{\partial \overline{c}_i(\xi',\ 1,\ \tau)}{\partial \tau_d\left(\overline{\xi}\right)} = 0\ \forall \overline{\xi} \in ET \setminus ET\left(\xi'\right).$$

Therefore,

$$g_{\overline{c}_i(\xi',\ 1,\ \tau)} = \frac{1}{\overline{c}_i(\xi',\ 1,\ \tau)} \cdot \frac{\partial \overline{c}_i(\xi',\ 1,\ \tau)}{\partial \tau_d\left(\overline{\xi}\right)} = 0\ \forall \overline{\xi} \in ET \setminus ET\left(\xi'\right).$$

We know that ξ is the initial node of the event tree ET. Thus, we can conclude by equation (3) from the previous Lemma 2.2.2. that

$$\frac{\partial \overline{\pi}(\xi',\ \tau)}{\partial \tau_d\left(\xi\right)} = \overline{\pi}(\xi', \tau) \cdot rr_i(\overline{c}_i(\xi,\ 1,\ \tau)) \cdot g_{\overline{c}_i(\xi,\ 1,\ \tau)}. \tag{3'}$$

Also, by equation (2)

$$\frac{\partial \overline{q}(\xi,\ \tau)}{\partial \tau_d(\xi)} = \underbrace{\sum_{\xi' \in ET^+(\xi)} \frac{\partial \overline{\pi}(\xi',\ \tau)}{\partial \tau_d\left(\xi\right)} \cdot \left(1 - \tau_d\left(\xi'\right)\right) \cdot d\left(\xi',\ \tau_d\right) +}_{Changes\ in\ the\ Stochastic\ Discount\ \overline{\pi}(\xi',\ \tau)\ Factor\ for\ \xi'}$$

$$\underbrace{+ \sum_{\xi' \in ET^+(\xi)} \overline{\pi}(\xi',\ \tau) \cdot \left[\frac{\partial d\left(\xi',\ \tau_d\right)}{\partial \tau_d\left(\xi\right)} \cdot \left(1 - \tau_d\left(\xi'\right)\right) - d\left(\xi',\ \tau_d\right) \cdot \frac{\partial \tau_d\left(\xi'\right)}{\partial \tau_d\left(\xi\right)}\right]}_{Changes\ in\ After-tax\ Dividends\ (1-\tau_d(\xi')) \cdot d(\xi',\ \tau_d)} \forall \xi \in ET.$$

Substituting (3') into the previous equation and taking into consideration that by the assumption of the Theorem

$$\frac{\partial \tau_d(\xi')}{\partial \tau_d(\xi)} = \frac{\partial d(\xi',\ \tau_d)}{\partial \tau_d(\xi)} = 0\ \forall\left(\xi,\ \xi'\right) \in ET \times \left[ET \setminus \{\xi\}\right].$$

we get

$$\frac{\partial \overline{q}(\xi,\,\tau)}{\partial \tau_d(\xi)} = \sum_{\xi' \in ET^+(\xi)} rr_i(\overline{c}_i(\xi,\,1,\,\tau)) \cdot g_{\overline{c}_i(\xi,\,1,\,\tau)} \cdot \overline{\pi}(\xi',\,\tau) \cdot (1 - \tau_d\,(\xi')) \cdot d\,(\xi',\;\tau_d) =$$
$$= rr_i(\overline{c}_i(\xi,\,1,\,\tau)) \cdot g_{\overline{c}_i(\xi,\,1,\,\tau)} \cdot \sum_{\xi' \in ET^+(\xi)} \overline{\pi}(\xi',\,\tau) \cdot (1 - \tau_d\,(\xi')) \cdot d\,(\xi',\;\tau_d) =$$
$$= \overline{q}\,(\xi,\;\tau) \cdot rr_i(\overline{c}_i\,(\xi,\;1,\;\tau)) \cdot g_{\overline{c}_i(\xi,\,1,\,\tau)}\;.$$

Therefore,

$$\boxed{\frac{\partial \overline{q}(\xi,\,\tau)}{\partial \tau_d(\xi)} = \overline{q}\,(\xi,\;\tau) \cdot rr_i(\overline{c}_i\,(\xi,\;1,\;\tau)) \cdot g_{\overline{c}_i(\xi,\,1,\,\tau)},} \tag{4}$$

where $rr_i(c) = -\left[\frac{u_i^{"}(c) \cdot c}{u_i'(c)}\right]$ is the coefficient of relative risk aversion of an agent $i \in I$,

$$g_{\overline{c}_i(\xi,\,1,\,\tau)} = \frac{1}{\overline{c}_i(\xi,\,1,\,\tau)} \cdot \frac{\partial \overline{c}_i(\xi,\,1,\,\tau)}{\partial \tau_d(\xi)}$$

and

$$\boxed{sign \boxed{\frac{\partial \overline{q}(\xi,\,\tau)}{\partial \tau_d(\xi)}} = sign \boxed{\frac{\partial \overline{c}_i(\xi,\,1,\,\tau)}{\partial \tau_d(\xi)}}\ \forall\,(\xi,\;i) \in ET \times I.} \blacksquare$$

PROOF OF COROLLARY 2.2.4: By Assumption D1, the total supply of assets is given by $\delta = \{\delta(k)\}_{k \in K} \in \mathbb{R}_{++}^{|K|}$. Then, since all agents are identical we have

$$\overline{c}_i(\xi,\,1,\,\tau) = (1 - \tau_e(\xi,\;1)) \cdot e(\xi,\,1,\,\tau_e) + \sum_{k \in K} (1 - \tau_d(\xi,\,k)) \cdot d(\xi,\,k,\,\tau_d) \cdot \frac{\delta(k)}{|I|}$$

$\forall(\xi,\,i) \in ET \times I.$

All agents have zero initial endowments of the numeraire good 1, i.e.,

$$e_i(\xi,\,1,\,\tau_{e_i}) = 0\ \forall\,(i,\;\xi) \in I \times ET.$$

Also, all assets' dividends are taxed identically, i.e.,

$$\tau_d(\xi,\,k) = \tau_d(\xi,\,\overline{k})\ \forall\,(\xi,\;k) \in ET \times K.$$

Thus, we have that

$$\overline{c}_i(\xi,\,1,\,\tau) = (1 - \tau_d(\xi,\,\overline{k})) \cdot \sum_{k \in K} d(\xi,\,k,\,\tau_d) \cdot \frac{\delta(k)}{|I|}\ \forall(\xi,\,i) \in ET \times I.$$

Also, by the assumption of the Corollary

$$\frac{\partial d(\xi',\,\tau_d)}{\partial \tau_d(\xi)} = 0\ \forall(\xi,\,\xi') \in ET \times ET.$$

Therefore,

$$\frac{\partial \overline{c}_i(\xi,\,1,\,\tau)}{\partial \tau_d(\xi,\,\overline{k})} = -\sum_{k \in K} d(\xi,\,k,\,\tau_d) \cdot \frac{\delta(k)}{|I|}\ \forall(\xi,\,i) \in ET \times I.$$

So we can conclude that

$$g_{\overline{c}_i(\xi,\,1,\,\tau)} = \frac{1}{\overline{c}_i(\xi,\,1,\,\tau)} \cdot \frac{\partial \overline{c}_i(\xi,\,1,\,\tau)}{\partial \tau_d\big(\xi,\,\overline{k}\big)} = \frac{-\sum_{k \in K} d(\xi,\,k,\,\tau_d) \cdot \frac{\delta(k)}{|I|}}{(1 - \tau_d(\xi,\,\overline{k})) \cdot \sum_{k \in K} d(\xi,\,k,\,\tau_d) \cdot \frac{\delta(k)}{|I|}} = -\frac{1}{(1 - \tau_d(\xi,\,\overline{k}))}\ \forall \xi \in ET.$$

Therefore, by (4) we obtain

$$\frac{\partial \overline{q}(\xi,\,\tau)}{\partial \tau_d(\xi)} = -\overline{q}\,(\xi,\ \tau) \cdot rr_i(\overline{c}_i\,(\xi,\ 1,\ \tau)) \cdot \frac{1}{(1-\tau_d(\xi,\,\overline{k}))}\ \forall(\xi,\,i) \in ET \times I.$$

Since all identical agents exhibit CRRA, i.e.,

$$rr_i(c) = -\left[\frac{u_i^{"}(c)\cdot c}{u_i'(c)}\right] = a\ \forall i \in I,$$

we have that

$$\frac{\partial \overline{q}(\xi,\,\tau)}{\partial \tau_d(\xi)} = -\overline{q}\,(\xi,\ \tau) \cdot a \cdot \frac{1}{(1-\tau_d(\xi,\,\overline{k}))}\ \forall \xi \in ET.$$

Hence,

$$E_{q(\xi,\,\tau),\,1-\tau_d(\xi,\,\overline{k})} = \frac{\frac{1}{\overline{q}(\xi,\,\tau)}\frac{\partial \overline{q}(\xi,\ \tau)}{\partial \tau_d(\xi,\,\overline{k})}}{\frac{1}{(1-\tau_d(\xi,\,\overline{k}))}\frac{\partial(1-\tau_d(\xi,\,\overline{k}))}{\partial \tau_d(\xi,\,\overline{k})}} = \frac{-a\cdot\frac{1}{(1-\tau_d(\xi,\,\overline{k}))}}{-\frac{1}{(1-\tau_d(\xi,\,\overline{k}))}} = a\ \forall \xi \in ET.$$

So

$$\boxed{E_{q(\xi,\,\tau),\,1-\tau_d(\xi,\,\overline{k})} = a\ \forall \xi \in ET.}$$

PROOF OF COROLLARY 2.2.5: We know that

$$R(\xi',\,\tau) = \frac{q(\xi',\,\tau)+d(\xi',\,\tau_d)}{q(\xi,\,\tau)},$$

where $\xi' \in \xi^+$. Differentiating $R(\xi',\,\tau)$ with respect to $\tau_d(\xi)$ we obtain

$$\frac{\partial R(\xi',\,\tau)}{\partial \tau_d(\xi)} = -\frac{q(\xi',\,\tau)+d(\xi',\,\tau_d)}{q^2(\xi,\,\tau)} \cdot \frac{\partial \overline{q}(\xi,\,\tau)}{\partial \tau_d(\xi)}.$$

We also know that

$$ATR(\xi',\,\tau) = \frac{q(\xi',\,\tau)+(1-\tau_d(\xi'))\cdot d(\xi',\,\tau_d)}{q(\xi,\,\tau)},$$

where $\xi' \in \xi^+$. Differentiating $ATR(\xi',\,\tau)$ with respect to $\tau_d(\xi)$ we obtain

$$\frac{\partial ATR(\xi',\,\tau)}{\partial \tau_d(\xi)} = -\frac{q(\xi',\,\tau)+(1-\tau_d(\xi'))\cdot d(\xi',\,\tau_d)}{q^2(\xi,\,\tau)} \cdot \frac{\partial \overline{q}(\xi,\,\tau)}{\partial \tau_d(\xi)}.$$

Therefore,

$$\boxed{sign\left[\frac{\partial R(\xi',\,\tau)}{\partial \tau_d(\xi)}\right] = sign\left[\frac{\partial ATR(\xi',\,\tau)}{\partial \tau_d(\xi)}\right] = -sign\left[\frac{\partial q(\xi,\,\tau)}{\partial \tau_d(\xi)}\right]\ \forall\,(\xi,\ \xi') \in ET \times \xi^+.}\ \blacksquare$$

PROOF OF THEOREM 2.2.6: Let ξ be the initial node of the event tree ET. Clearly,

$$\overline{\pi}(\xi',\,\tau) = b_i^{T(\xi')} \cdot \left(\frac{\overline{c}_i(\xi',\,1,\,\tau)}{\overline{c}_i(\xi,\,1,\,\tau)}\right)^{-a} \cdot \Pr(\xi')\ \forall \xi' \in ET^+(\xi).$$

By the assumption of the Theorem

$$\frac{\overline{c}_i(\xi',\,1,\,\tau)}{\overline{c}_i(\xi,\,1,\,\tau)} = \frac{\sum_{i\in I}\left(1-\tau_{e_i}(\xi',\,1)\right)\cdot e_i(\xi',\,1,\,\tau_{e_i})+\sum_{k\in K}(1-\tau_d(\xi',\,k))\cdot d(\xi',\,k,\,\tau_d)\cdot\delta(k)}{\sum_{i\in I}\left(1-\tau_{e_i}(\xi,\,1)\right)\cdot e_i(\xi,\,1,\,\tau_{e_i})+\sum_{k\in K}(1-\tau_d(\xi,\,k))\cdot d(\xi,\,k,\,\tau_d)\cdot\delta(k)}$$

$\forall\,(\xi,\ \xi',\ \tau,\ i) \in ET \times ET \times O_{\overline{\tau}} \times I$, where $\delta(k)$ is the total number of outstanding shares of asset $k \in K$. Therefore,

$$\overline{\pi}(\xi',\,\tau) = b_i^{T(\xi')} \cdot \left(\frac{\sum_{i\in I}\big(1-\tau_{e_i}(\xi',\,1)\big)\cdot e_i(\xi',\,1,\,\tau_{e_i}) + \sum_{k\in K}(1-\tau_d(\xi',\,k))\cdot d(\xi',\,k,\,\tau_d)\cdot\delta(k)}{\sum_{i\in I}\big(1-\tau_{e_i}(\xi,\,1)\big)\cdot e_i(\xi,\,1,\,\tau_{e_i}) + \sum_{k\in K}(1-\tau_d(\xi,\,k))\cdot d(\xi,\,k,\,\tau_d)\cdot\delta(k)} \right)^{-a} \cdot \Pr(\xi')$$

$\forall \xi' \in ET^+(\xi)$. Fix $\overline{\xi} \in ET^+(\xi)$. Then clearly,

$$\frac{\partial \overline{\pi}(\xi',\,\tau)}{\partial \tau_d\big(\overline{\xi},\,k\big)} = 0\ \forall \xi' \in ET^+(\xi) \setminus \big\{\overline{\xi}\big\}.$$

If $\xi' = \overline{\xi}$, then

$$\frac{\partial \overline{\pi}(\overline{\xi},\,\tau)}{\partial \tau_d\big(\overline{\xi},\,k\big)} = -a \cdot b_i^{T(\overline{\xi})} \cdot \left(\frac{\overline{c}_i(\overline{\xi},\,1,\,\tau)}{\overline{c}_i(\xi,\,1,\,\tau)}\right)^{-a-1} \cdot \Pr(\overline{\xi}) \cdot$$
$$\cdot \left(\frac{\left[\frac{\partial d(\overline{\xi},\,k,\,\tau_d)}{\partial \tau_d(\overline{\xi},\,k)}\cdot\big(1-\tau_d\big(\overline{\xi},\,k\big)\big) - d(\overline{\xi},\,k,\,\tau_d)\right]\cdot\delta(k)}{\sum_{i\in I}\big(1-\tau_{e_i}(\xi,\,1)\big)\cdot e_i(\xi,\,1,\,\tau_{e_i}) + \sum_{k\in K}(1-\tau_d(\xi,\,k))\cdot d(\xi,\,k,\,\tau_d)\cdot\delta(k)} \right) =$$
$$= -a \cdot \underbrace{b_i^{T(\overline{\xi})} \cdot \left(\frac{\overline{c}_i\big(\overline{\xi},\ 1,\ \tau\big)}{\overline{c}_i(\xi,\ 1,\ \tau)}\right)^{-a} \cdot \Pr\big(\overline{\xi}\big)}_{\overline{\pi}(\overline{\xi},\,\tau)} \cdot \left(\frac{\overline{c}_i(\overline{\xi},\,1,\,\tau)}{\overline{c}_i(\xi,\,1,\,\tau)}\right)^{-1} \cdot$$
$$\cdot \left(\frac{\left[\frac{\partial d(\overline{\xi},\,k,\,\tau_d)}{\partial \tau_d(\overline{\xi},\,k)}\cdot\big(1-\tau_d\big(\overline{\xi},\,k\big)\big) - d(\overline{\xi},\,k,\,\tau_d)\right]\cdot\delta(k)}{\sum_{i\in I}\big(1-\tau_{e_i}(\xi,\,1)\big)\cdot e_i(\xi,\,1,\,\tau_{e_i}) + \sum_{k\in K}(1-\tau_d(\xi,\,k))\cdot d(\xi,\,k,\,\tau_d)\cdot\delta(k)} \right) =$$
$$= -a \cdot \overline{\pi}(\overline{\xi},\,\tau) \cdot \left(\frac{\sum_{i\in I}\big(1-\tau_{e_i}(\xi,\,1)\big)\cdot e_i(\xi,\,1,\,\tau_{e_i}) + \sum_{k\in K}(1-\tau_d(\xi,\,k))\cdot d(\xi,\,k,\,\tau_d)\cdot\delta(k)}{\sum_{i\in I}\big(1-\tau_{e_i}(\overline{\xi},\,1)\big)\cdot e_i(\overline{\xi},\,1,\,\tau_{e_i}) + \sum_{k\in K}\big(1-\tau_d(\overline{\xi},\,k)\big)\cdot d(\overline{\xi},\,k,\,\tau_d)\cdot\delta(k)} \right) \cdot$$
$$\cdot \left(\frac{\left[\frac{\partial d(\overline{\xi},\,k,\,\tau_d)}{\partial \tau_d(\overline{\xi},\,k)}\cdot\big(1-\tau_d\big(\overline{\xi},\,k\big)\big) - d(\overline{\xi},\,k,\,\tau_d)\right]\cdot\delta(k)}{\sum_{i\in I}\big(1-\tau_{e_i}(\xi,\,1)\big)\cdot e_i(\xi,\,1,\,\tau_{e_i}) + \sum_{k\in K}(1-\tau_d(\xi,\,k))\cdot d(\xi,\,k,\,\tau_d)\cdot\delta(k)} \right) =$$
$$= a \cdot \overline{\pi}(\overline{\xi},\,\tau) \cdot \frac{\left[d(\overline{\xi},\,k,\,\tau_d) - \frac{\partial d(\overline{\xi},\,k,\,\tau_d)}{\partial \tau_d(\overline{\xi},\,k)}\cdot\big(1-\tau_d\big(\overline{\xi},\,k\big)\big)\right]\cdot\delta(k)}{\sum_{i\in I}\big(1-\tau_{e_i}(\overline{\xi},\,1)\big)\cdot e_i(\overline{\xi},\,1,\,\tau_{e_i}) + \sum_{k\in K}\big(1-\tau_d(\overline{\xi},\,k)\big)\cdot d(\overline{\xi},\,k,\,\tau_d)\cdot\delta(k)}.$$

Hence,

$$\frac{\partial \overline{\pi}(\overline{\xi},\,\tau)}{\partial \tau_d\big(\overline{\xi},\,k\big)} = a \cdot \overline{\pi}(\overline{\xi},\,\tau) \cdot \frac{\left[d(\overline{\xi},\,k,\,\tau_d) - \frac{\partial d(\overline{\xi},\,k,\,\tau_d)}{\partial \tau_d(\overline{\xi},\,k)}\cdot\big(1-\tau_d\big(\overline{\xi},\,k\big)\big)\right]\cdot\delta(k)}{\sum_{i\in I}\big(1-\tau_{e_i}(\overline{\xi},\,1)\big)\cdot e_i(\overline{\xi},\,1,\,\tau_{e_i}) + \sum_{k\in K}\big(1-\tau_d(\overline{\xi},\,k)\big)\cdot d(\overline{\xi},\,k,\,\tau_d)\cdot\delta(k)}.$$

Set

$$\Pi(\overline{\xi},\,k,\,\tau) = \frac{\left[d(\overline{\xi},\,k,\,\tau_d) - \frac{\partial d(\overline{\xi},\,k,\,\tau_d)}{\partial \tau_d(\overline{\xi},\,k)}\cdot\big(1-\tau_d\big(\overline{\xi},\,k\big)\big)\right]\cdot\delta(k)}{\sum_{i\in I}\big(1-\tau_{e_i}(\overline{\xi},\,1)\big)\cdot e_i(\overline{\xi},\,1,\,\tau_{e_i}) + \sum_{k\in K}\big(1-\tau_d(\overline{\xi},\,k)\big)\cdot d(\overline{\xi},\,k,\,\tau_d)\cdot\delta(k)}.$$

Then

$$\frac{\partial\overline{\pi}(\overline{\xi},\,\tau)}{\partial\tau_d\left(\overline{\xi},\,k\right)} = a\cdot\overline{\pi}(\overline{\xi},\,\tau)\cdot\Pi(\overline{\xi},\,k,\,\tau).$$

We also know that

$$\frac{\partial\overline{q}(\xi,\,k,\,\tau)}{\partial\tau_d\left(\overline{\xi},\,k\right)} = \underbrace{\sum_{\xi'\in ET^{+}(\xi)}\frac{\partial\overline{\pi}(\xi',\ \tau)}{\partial\tau_d\left(\overline{\xi},\ k\right)}\cdot\left(1-\tau_d\left(\xi',\ k\right)\right)\cdot d\left(\xi',\ k,\ \tau_d\right)+}_{Changes\ in\ the\ Stochastic\ Discount\ \overline{\pi}(\xi',\,\tau)\ Factor\ for\ \xi'}$$
$$+\underbrace{\sum_{\xi'\in ET^{+}(\xi)}\overline{\pi}(\xi',\ \tau)\cdot\left[\frac{\partial d\left(\xi',\ k,\ \tau_d\right)}{\partial\tau_d\left(\overline{\xi},\ k\right)}\cdot\left(1-\tau_d\left(\xi',\ k\right)\right)-d\left(\xi',\ k,\ \tau_d\right)\cdot\frac{\partial\tau_d\left(\xi',\ k\right)}{\partial\tau_d\left(\overline{\xi},\ k\right)}\right]}_{Changes\ in\ After-tax\ Dividends\ (1-\tau_d(\xi',\,k))\cdot d(\xi',\,k,\,\tau_d)}\ \forall\overline{\xi}\in ET^{+}(\xi).$$

Substituting expression for $\frac{\partial\overline{\pi}(\overline{\xi},\,\tau)}{\partial\tau_d\left(\overline{\xi},\,k\right)}$ into the previous equation and taking into consideration that by Assumptions D4 and D5

$$\frac{\partial\tau_d(\xi',\,k)}{\partial\tau_d(\xi,\,k)} = \frac{\partial d(\xi',\,k,\,\tau_d)}{\partial\tau_d(\xi,\,k)} = 0\ \forall\left(\xi,\ \xi'\right)\in ET\times\left[ET\setminus\{\xi\}\right],$$

we get

$$\frac{\partial\overline{q}(\xi,\,k,\,\tau)}{\partial\tau_d\left(\overline{\xi},\,k\right)} = a\cdot\overline{\pi}(\overline{\xi},\,\tau)\cdot\Pi(\overline{\xi},\,k,\,\tau)\cdot\left(1-\tau_d\left(\overline{\xi},\ k\right)\right)\cdot d\left(\overline{\xi},\ k,\ \tau_d\right)+$$
$$+\overline{\pi}(\overline{\xi},\,\tau)\cdot\left[\frac{\partial d\left(\overline{\xi},\,k,\,\tau_d\right)}{\partial\tau_d\left(\overline{\xi},\,k\right)}\cdot\left(1-\tau_d\left(\overline{\xi},\ k\right)\right)-d\left(\overline{\xi},\ k,\ \tau_d\right)\right].$$

Therefore,

$$a > \frac{d\left(\overline{\xi},\,k,\,\tau_d\right)-\frac{\partial d\left(\overline{\xi},\,k,\,\tau_d\right)}{\partial\tau_d\left(\overline{\xi},\,k\right)}\cdot\left(1-\tau_d\left(\overline{\xi},\,k\right)\right)}{\Pi(\overline{\xi},\,k,\,\tau)\cdot\left(1-\tau_d\left(\overline{\xi},\,k\right)\right)\cdot d\left(\overline{\xi},\,k,\,\tau_d\right)} =$$
$$= \frac{d\left(\overline{\xi},\,k,\,\tau_d\right)-\frac{\partial d\left(\overline{\xi},\,k,\,\tau_d\right)}{\partial\tau_d\left(\overline{\xi},\,k\right)}\cdot\left(1-\tau_d\left(\overline{\xi},\,k\right)\right)}{\left[\frac{\left[d\left(\overline{\xi},\,k,\,\tau_d\right)-\frac{\partial d\left(\overline{\xi},\,k,\,\tau_d\right)}{\partial\tau_d\left(\overline{\xi},\,k\right)}\cdot\left(1-\tau_d\left(\overline{\xi},\,k\right)\right)\right]\cdot\delta(k)}{\sum_{i\in I}\left(1-\tau_{e_i}(\overline{\xi},\,1)\right)\cdot e_i(\overline{\xi},\,1,\,\tau_{e_i})+\sum_{k\in K}\left(1-\tau_d(\overline{\xi},\,k)\right)\cdot d(\overline{\xi},\,k,\,\tau_d)\cdot\delta(k)}\right]\cdot\left(1-\tau_d\left(\overline{\xi},\,k\right)\right)\cdot d\left(\overline{\xi},\,k,\,\tau_d\right)} =$$
$$= \frac{1}{\left[\frac{\left(1-\tau_d\left(\overline{\xi},\,k\right)\right)\cdot d\left(\overline{\xi},\,k,\,\tau_d\right)\cdot\delta(k)}{\sum_{i\in I}\left(1-\tau_{e_i}(\overline{\xi},\,1)\right)\cdot e_i(\overline{\xi},1,\tau_{e_i})+\sum_{k\in K}\left(1-\tau_d(\overline{\xi},\,k)\right)\cdot d(\overline{\xi},k,\tau_d)\cdot\delta(k)}\right]} = B(\overline{\xi},\,k,\,\tau).$$

Therefore,

$$a > B(\overline{\xi},\,k,\,\tau) = \frac{1}{\left[\frac{\left(1-\tau_d\left(\overline{\xi},\,k\right)\right)\cdot d\left(\overline{\xi},\,k,\,\tau_d\right)\cdot\delta(k)}{\sum_{i\in I}\left(1-\tau_{e_i}(\overline{\xi},\,1)\right)\cdot e_i(\overline{\xi},\,1,\,\tau_{e_i})+\sum_{k\in K}\left(1-\tau_d(\overline{\xi},\,k)\right)\cdot d(\overline{\xi},\,k,\,\tau_d)\cdot\delta(k)}\right]} \geqslant 1.$$

Thus,

$$\boxed{sign\left[\frac{\partial\overline{q}(\xi,\,k,\,\tau)}{\partial\tau_d\left(\overline{\xi},\,k\right)}\right] = sign\left[a-B(\overline{\xi},\ k,\ \tau)\right]\ \forall\overline{\xi}\in ET^{+}(\xi).}\ \blacksquare$$

PROOF OF THEOREM 2.2.7: By Assumption D1, the total supply of assets is given by $\delta = \{\delta(k)\}_{k\in K} \in \mathbb{R}^{|K|}_{++}$. Then, since all agents are identical we have

$$\overline{c}_i(\xi,\, 1,\, \tau) = (1-\tau_e(\xi,\ 1)) \cdot e(\xi,\, 1,\, \tau_e) + \sum_{k\in K}(1-\tau_d(\xi,\, k)) \cdot d(\xi,\, k,\, \tau_d) \cdot \tfrac{\delta(k)}{|I|}$$

$\forall(\xi,\, i) \in ET \times I$. Fix $\overline{\xi} \in ET^{+}(\xi)$. Therefore,

$$\tfrac{\partial \overline{c}_i(\xi',\, 1,\, \tau)}{\partial \tau_d\left(\overline{\xi},\, k\right)} =$$

$$= \begin{cases} 0 & \forall \xi' \in ET \setminus \{\overline{\xi}\} \\ \left[-d(\overline{\xi}, k, \tau_d) + \frac{\partial d\left(\overline{\xi},\, k,\, \tau_d\right)}{\partial \tau_d\left(\overline{\xi},\, k\right)} \cdot (1-\tau_d(\overline{\xi}, k))\right] \cdot \frac{\delta(k)}{|I|} & \text{if } \xi' = \overline{\xi} \end{cases}.$$

So we can conclude that

$$g_{\overline{c}(\xi',\, 1,\, \tau)} = \tfrac{1}{\overline{c}(\xi',\, 1,\, \tau)} \cdot \tfrac{\partial \overline{c}(\xi',\, 1,\, \tau)}{\partial \tau_d\left(\overline{\xi},\, k\right)} =$$

$$= \begin{cases} 0 & \forall \xi' \in ET \setminus \{\overline{\xi}\} \\ \frac{\left[-d(\overline{\xi},\, k,\, \tau_d) + \frac{\partial d(\overline{\xi},\, k,\, \tau_d)}{\partial \tau_d(\overline{\xi},\, k)} \cdot (1-\tau_d(\overline{\xi},\, k))\right] \cdot \frac{\delta(k)}{|I|}}{\overline{c}(\overline{\xi},\, 1,\, \tau)} & \text{if } \xi' = \overline{\xi} \end{cases}$$

and by equation (3)

$$\tfrac{\partial \overline{\pi}(\xi',\, \tau)}{\partial \tau_d\left(\overline{\xi},\, \tau_d,\, k\right)} =$$

$$= \begin{cases} 0 & \forall \xi' \in ET \setminus \{\overline{\xi}\} \\ -\overline{\pi}(\overline{\xi},\, \tau) \cdot rr(\overline{c}(\overline{\xi},\, 1,\, \tau)) \cdot g_{\overline{c}(\overline{\xi},\, 1,\, \tau)} & \text{if } \xi' = \overline{\xi} \end{cases} \tag{3”}$$

We also know that

$$\tfrac{\partial \overline{q}(\xi,\, k,\, \tau)}{\partial \tau_d\left(\overline{\xi},\, k\right)} = \underbrace{\sum_{\xi' \in ET^{+}(\xi)} \frac{\partial \overline{\pi}(\xi',\ \tau)}{\partial \tau_d\left(\overline{\xi},\ k\right)} \cdot (1-\tau_d\left(\xi',\ k\right)) \cdot d\left(\xi',\ k,\ \tau_d\right) +}_{Changes\ in\ the\ Stochastic\ Discount\ \overline{\pi}(\xi',\, \tau)\ Factor\ for\ \xi'}$$

$$+ \underbrace{\sum_{\xi' \in ET^{+}(\xi)} \overline{\pi}(\xi',\ \tau) \cdot \left[\frac{\partial d\left(\xi',\ k,\ \tau_d\right)}{\partial \tau_d\left(\overline{\xi},\ k\right)} \cdot (1-\tau_d\left(\xi',\ k\right)) - d\left(\xi',\ k,\ \tau_d\right) \cdot \frac{\partial \tau_d\left(\xi',\ k\right)}{\partial \tau_d\left(\overline{\xi},\ k\right)}\right]}_{Changes\ in\ After-tax\ Dividends\ (1-\tau_d(\xi',\, k)) \cdot d(\xi',\, k,\, \tau_d)}$$

$\forall \overline{\xi} \in ET^{+}(\xi)$.

Substituting (3”) into the previous equation and taking into consideration that by Assumptions D4 and D5

$$\tfrac{\partial \tau_d(\xi',\, k)}{\partial \tau_d(\xi,\, k)} = \tfrac{\partial d(\xi',\, k,\, \tau_d)}{\partial \tau_d(\xi,\, k)} = 0 \ \forall \left(\xi,\ \xi'\right) \in ET \times \left[ET \setminus \{\xi\}\right],$$

we get

$$\tfrac{\partial \overline{q}(\xi,\, k,\, \tau)}{\partial \tau_d\left(\overline{\xi},\, k\right)} = -\overline{\pi}(\overline{\xi}, \tau) \cdot rr(\overline{c}(\overline{\xi}, 1, \tau)) \cdot g_{\overline{c}(\overline{\xi},\, 1,\, \tau)} \cdot \left(1-\tau_d\left(\overline{\xi},\ k\right)\right) \cdot d\left(\overline{\xi},\ k,\ \tau_d\right) +$$
$$+\overline{\pi}(\overline{\xi},\, \tau) \cdot \left[\tfrac{\partial d\left(\overline{\xi},\, k,\, \tau_d\right)}{\partial \tau_d\left(\overline{\xi},\, k\right)} \cdot \left(1-\tau_d\left(\overline{\xi},\ k\right)\right) - d\left(\overline{\xi},\ k,\ \tau_d\right)\right] \ \forall \overline{\xi} \in ET^{+}(\xi).$$

Let

$$\frac{\partial \overline{q}(\xi,\,k,\,\tau)}{\partial \tau_d\left(\overline{\xi},\,k\right)} > 0.$$

We know that

$$g_{\overline{c}(\overline{\xi},\,1,\,\tau)} = \frac{1}{\overline{c}(\overline{\xi},\,1,\,\tau)} \cdot \frac{\partial \overline{c}(\overline{\xi},\,1,\,\tau)}{\partial \tau_d\left(\overline{\xi},\,k\right)} < 0.$$

Therefore,

$$rr(\overline{c}(\overline{\xi},\,1,\,\tau)) > \frac{d\left(\overline{\xi},\,k,\,\tau_d\right) - \frac{\partial d\left(\overline{\xi},\,k,\,\tau_d\right)}{\partial \tau_d\left(\overline{\xi},\,k\right)} \cdot \left(1-\tau_d\left(\overline{\xi},\,k\right)\right)}{-g_{\overline{c}(\overline{\xi},\,1,\,\tau)} \cdot \left(1-\tau_d\left(\overline{\xi},\,k\right)\right) \cdot d\left(\overline{\xi},\,k,\,\tau_d\right)} =$$

$$= \frac{d\left(\overline{\xi},\,k,\,\tau_d\right) - \frac{\partial d\left(\overline{\xi},\,k,\,\tau_d\right)}{\partial \tau_d\left(\overline{\xi},\,k\right)} \cdot \left(1-\tau_d\left(\overline{\xi},\,k\right)\right)}{\frac{\left[d(\overline{\xi},\,k,\,\tau_d) - \frac{\partial d\left(\overline{\xi},\,k,\,\tau_d\right)}{\partial \tau_d\left(\overline{\xi},\,k\right)} \cdot (1-\tau_d(\overline{\xi},\,k))\right] \cdot \frac{\delta(k)}{|I|}}{\left(1-\tau_e(\overline{\xi},\,1)\right) \cdot e(\overline{\xi},\,1,\,\tau_e) + \sum_{k\in K} (1-\tau_d(\overline{\xi},\,k)) \cdot d(\overline{\xi},\,k,\,\tau_d) \cdot \frac{\delta(k)}{|I|}} \cdot \left(1-\tau_d\left(\overline{\xi},\,k\right)\right) \cdot d\left(\overline{\xi},\,k,\,\tau_d\right)} =$$

$$= \frac{1}{\left[\frac{\left(1-\tau_d(\overline{\xi},\,k)\right) \cdot d(\overline{\xi},\,k,\,\tau_d) \cdot \frac{\delta(k)}{|I|}}{\left(1-\tau_e(\overline{\xi},\,1)\right) \cdot e(\overline{\xi},\,1,\,\tau_e) + \sum_{k\in K} (1-\tau_d(\overline{\xi},\,k)) \cdot d(\overline{\xi},\,k,\,\tau_d) \cdot \frac{\delta(k)}{|I|}}\right]} = B(\overline{\xi},\,k,\,\tau).$$

Therefore,

$$rr(\overline{c}(\overline{\xi},\,1,\,\tau)) > B(\overline{\xi},\,k,\,\tau) = \frac{1}{\left[\frac{\left(1-\tau_d(\overline{\xi},\,k)\right) \cdot d(\overline{\xi},\,k,\,\tau_d) \cdot \frac{\delta(k)}{|I|}}{\left(1-\tau_e(\overline{\xi},\,1)\right) \cdot e(\overline{\xi},\,1,\,\tau_e) + \sum_{k\in K} (1-\tau_d(\overline{\xi},\,k)) \cdot d(\overline{\xi},\,k,\,\tau_d) \cdot \frac{\delta(k)}{|I|}}\right]} \geqslant 1.$$

Thus,

$$\boxed{sign \left[\frac{\partial \overline{q}(\xi,\,\tau)}{\partial \tau_d\left(\overline{\xi}\right)}\right] = sign\left[rr(\overline{c}(\overline{\xi},\;1,\;\tau)) - B(\overline{\xi},\;k,\;\tau)\right]\ \forall \overline{\xi} \in ET^{+}(\xi).} \blacksquare$$

PROOF OF COROLLARY 2.2.8: By Assumption D1, the total supply of assets is given by $\delta = \{\delta(k)\}_{k\in K} \in \mathbb{R}^{|K|}_{++}$. Then, since all agents are identical we have

$$\overline{c}(\xi,\,1,\,\tau) = (1-\tau_e(\xi,\;1)) \cdot e(\xi,\,1,\,\tau_e) + \sum_{k\in K} (1-\tau_d(\xi,\,k)) \cdot d(\xi,\,k,\,\tau_d) \cdot \tfrac{\delta(k)}{|I|}\ \forall \xi \in ET.$$

When all agents are the same and have zero initial endowments of the numeraire *good* 1, it means

$$\overline{c}(\xi,\,1,\,\tau) = \sum_{k\in K} (1-\tau_d(\xi,\,k)) \cdot d(\xi,\,k,\,\tau_d) \cdot \tfrac{\delta(k)}{|I|}\ \forall \xi \in ET.$$

Also, all assets' dividends are taxed identically, i.e.,

$$\tau_d(\xi,\,k) = \tau_d(\xi,\,\overline{k})\ \forall\,(\xi,\;k) \in ET \times K.$$

Thus, we have that

$$\overline{c}(\xi,\,1,\,\tau)=\sum_{k\in K}(1-\tau_d(\xi,\,\overline{k}))\cdot d(\xi,\,k,\,\tau_d)\cdot\tfrac{\delta(k)}{|I|}\ \forall\xi\in ET.$$

Also, by the assumption of the Corollary

$$\tfrac{\partial d}{\partial\tau_d}=0.$$

Fix $\overline{\xi}\in ET^{+}(\xi)$. Therefore,

$$\tfrac{\partial\overline{c}(\overline{\xi},\,1,\,\tau)}{\partial\tau_d(\overline{\xi},\,\overline{k})}=\begin{cases}0 & \forall\xi'\in ET\setminus\{\overline{\xi}\}\\ -\sum_{k\in K}d(\overline{\xi},\,k,\,\tau_d)\cdot\tfrac{\delta(k)}{|I|}. & \text{if }\xi'=\overline{\xi}\end{cases}.$$

So we can conclude that

$$g_{\overline{c}(\overline{\xi},\,1,\,\tau)}=\tfrac{1}{\overline{c}(\overline{\xi},\,1,\,\tau)}\cdot\tfrac{\partial\overline{c}(\overline{\xi},\,1,\,\tau)}{\partial\tau_d\left(\overline{\xi},\,\overline{k}\right)}=\begin{cases}0 & \forall\xi'\in ET\setminus\{\overline{\xi}\}\\ \dfrac{-\sum_{k\in K}d(\overline{\xi},\,k,\,\tau_d)\cdot\frac{\delta(k)}{|I|}}{\sum_{k\in K}(1-\tau_d(\overline{\xi},\,\overline{k})\cdot d(\overline{\xi},\,k,\,\tau_d)\cdot\frac{\delta(k)}{|I|}}=-\tfrac{1}{(1-\tau_d(\overline{\xi},\,\overline{k})} & \text{if }\xi'=\overline{\xi}\end{cases}.$$

Let

$$\tfrac{\partial\overline{q}(\xi,\,k,\,\tau)}{\partial\tau_d\left(\overline{\xi},\,k\right)}>0.$$

Then we know from Theorem 2.2.7. that

$$rr(\overline{c}(\overline{\xi},\,1,\,\tau))>\frac{d\left(\overline{\xi},\,k,\,\tau_d\right)-\frac{\partial d\left(\overline{\xi},\,\,k,\,\tau_d\right)}{\partial\tau_d\left(\overline{\xi},\,k\right)}\cdot\left(1-\tau_d\left(\overline{\xi},\,k\right)\right)}{-g_{\overline{c}(\overline{\xi},\,1,\,\tau)}\cdot\left(1-\tau_d\left(\overline{\xi},\,k\right)\right)\cdot d\left(\overline{\xi},\,k,\,\tau_d\right)}.$$

Substituting the expression for $g_{\overline{c}(\overline{\xi},\,1,\,\tau)}$ into the right-hand side of the above inequality and keeping in mind that $\frac{\partial d}{\partial\tau_d}=0$ we obtain

$$\frac{d\left(\overline{\xi},\,k,\,\tau_d\right)-\frac{\partial d\left(\overline{\xi},\,\,k,\,\tau_d\right)}{\partial\tau_d\left(\overline{\xi},\,k\right)}\cdot\left(1-\tau_d\left(\overline{\xi},\,k\right)\right)}{-g_{\overline{c}(\overline{\xi},\,1,\,\tau)}\cdot\left(1-\tau_d\left(\overline{\xi},\,k\right)\right)\cdot d\left(\overline{\xi},\,k,\,\tau_d\right)}=\frac{d\left(\overline{\xi},\,k,\,\tau_d\right)}{\left[\frac{1}{(1-\tau_d(\overline{\xi},\,\overline{k})}\right]\cdot\left(1-\tau_d\left(\overline{\xi},\,\overline{k}\right)\right)\cdot d\left(\overline{\xi},\,k,\,\tau_d\right)}=1.$$

Thus,

$$B(\overline{\xi},\,k,\,\tau)=1.$$

and

$$\boxed{sign\left[\frac{\partial\overline{q}(\xi,\,k,\,\tau)}{\partial\tau_d\left(\overline{\xi},\,\overline{\overline{k}}\right)}\right]=sign\left[rr(\overline{c}(\overline{\xi},\ 1,\ \tau))-1\right]\ \forall\overline{\xi}\in ET^{+}(\xi).}\ \blacksquare$$

Proofs for Comparative Statics of FM Equilibria with Respect to the Endowment Tax τ_{e_i}

PROOF OF THEOREM 2.3.4: Let ξ be the initial node of the event tree ET. Clearly,

$$\overline{\pi}(\xi',\,\tau)=b_i^{T(\xi')}\cdot\left(\tfrac{\overline{c}_i(\xi',\,1,\,\tau)}{\overline{c}_i(\xi,\,1,\,\tau)}\right)^{-a_i}\cdot\Pr(\xi')\ \forall\xi'\in ET^{+}(\xi).$$

By the assumption of the Theorem

$$\frac{\overline{c}_i(\xi',\,1,\,\tau)}{\overline{c}_i(\xi,\,1,\,\tau)} = \frac{\sum\limits_{i\in I}\Big(1-\tau_{e_i}(\xi',\,1)\Big)\cdot e_i(\xi',\,1,\,\tau_{e_i})+\sum\limits_{k\in K}\delta(k)\cdot(1-\tau_d(\xi',\,k))\cdot d(\xi',\,k,\,\tau_d)}{\sum\limits_{i\in I}\Big(1-\tau_{e_i}(\xi,\,1)\Big)\cdot e_i(\xi,\,1,\,\tau_{e_i})+\sum\limits_{k\in K}\delta(k)\cdot(1-\tau_d(\xi,\,k))\cdot d(\xi,\,k,\,\tau_d)}$$

$\forall\,(\xi,\ \xi',\ \tau,\ i) \in ET \times ET \times O_{\overline{\tau}} \times I$, where $\delta(k)$ is the total number of outstanding shares of asset $k \in K$. Therefore,

$$\overline{\pi}(\xi',\,\tau) = b_i^{T(\xi')} \cdot \left(\frac{\sum\limits_{i\in I}\Big(1-\tau_{e_i}(\xi',\,1)\Big)\cdot e_i(\xi',\,1,\,\tau_{e_i})+\sum\limits_{k\in K}\delta(k)\cdot(1-\tau_d(\xi',\,k))\cdot d(\xi',\,k,\,\tau_d)}{\sum\limits_{i\in I}\Big(1-\tau_{e_i}(\xi,\,1)\Big)\cdot e_i(\xi,\,1,\,\tau_{e_i})+\sum\limits_{k\in K}\delta(k)\cdot(1-\tau_d(\xi,\,k))\cdot d(\xi,\,k,\,\tau_d)}\right)^{-a_i} \cdot \Pr(\xi')$$

$\forall \xi' \in ET^+(\xi)$. Fix $\overline{\xi} \in ET^+(\xi)$. Then clearly,

$$\frac{\partial\overline{\pi}(\xi',\,\tau)}{\partial\tau_{e_i}\big(\overline{\xi},\,1\big)} = 0\ \forall \xi' \in ET^+(\xi) \setminus \big\{\overline{\xi}\big\}.$$

If $\xi' = \overline{\xi}$, then

$$\frac{\partial\overline{\pi}(\overline{\xi},\,\tau)}{\partial\tau_{e_i}\big(\overline{\xi},\,1\big)} = -a_i \cdot b_i^{T(\overline{\xi})} \cdot \left(\frac{\overline{c}_i(\overline{\xi},\,1,\,\tau)}{\overline{c}_i(\xi,\,1,\,\tau)}\right)^{-a_i-1} \cdot \Pr(\overline{\xi})\cdot$$

$$\cdot\left(\frac{-e_i(\overline{\xi},\,1,\,\tau_{e_i})}{\sum\limits_{i\in I}\Big(1-\tau_{e_i}(\xi,\,1)\Big)\cdot e_i(\xi,\,1,\,\tau_{e_i})+\sum\limits_{k\in K}\delta(k)\cdot(1-\tau_d(\xi,\,k))\cdot d(\xi,\,k,\,\tau_d)}\right) =$$

$$= -a_i \cdot \underbrace{b_i^{T(\overline{\xi})} \cdot \left(\frac{\overline{c}_i(\overline{\xi},\ 1,\ \tau)}{\overline{c}_i(\xi,\ 1,\ \tau)}\right)^{-a_i} \cdot \Pr(\overline{\xi})}_{\overline{\pi}(\overline{\xi},\,\tau)} \cdot \left(\frac{\overline{c}_i(\overline{\xi},\,1,\,\tau)}{\overline{c}_i(\xi,\,1,\,\tau)}\right)^{-1}\cdot$$

$$\cdot\left(\frac{-e_i(\overline{\xi},\,1,\,\tau_{e_i})}{\sum\limits_{i\in I}\Big(1-\tau_{e_i}(\xi,\,1)\Big)\cdot e_i(\xi,\,1,\,\tau_{e_i})+\sum\limits_{k\in K}\delta(k)\cdot(1-\tau_d(\xi,\,k))\cdot d(\xi,\,k,\,\tau_d)}\right) =$$

$$= -a_i \cdot \overline{\pi}(\overline{\xi},\,\tau) \cdot \left(\frac{\sum\limits_{i\in I}\Big(1-\tau_{e_i}(\xi,\,1)\Big)\cdot e_i(\xi,\,1,\,\tau_{e_i})+\sum\limits_{k\in K}\delta(k)\cdot(1-\tau_d(\xi,\,k))\cdot d(\xi,\,k,\,\tau_d)}{\sum\limits_{i\in I}\Big(1-\tau_{e_i}(\overline{\xi},\,1)\Big)\cdot e_i(\overline{\xi},\,1,\,\tau_{e_i})+\sum\limits_{k\in K}\delta(k)\cdot\Big(1-\tau_d(\overline{\xi},\,k)\Big)\cdot d(\overline{\xi},\,k,\,\tau_d)}\right)\cdot$$

$$\cdot\left(\frac{-e_i(\overline{\xi},\,1,\,\tau_{e_i})}{\sum\limits_{i\in I}\Big(1-\tau_{e_i}(\xi,\,1)\Big)\cdot e_i(\xi,\,1,\,\tau_{e_i})+\sum\limits_{k\in K}\delta(k)\cdot(1-\tau_d(\xi,\,k))\cdot d(\xi,\,k,\,\tau_d)}\right) =$$

$$= a_i \cdot \overline{\pi}(\overline{\xi},\,\tau) \cdot \frac{e_i(\overline{\xi},\,1,\,\tau_{e_i})}{\sum\limits_{i\in I}\Big(1-\tau_{e_i}(\overline{\xi},\,1)\Big)\cdot e_i(\overline{\xi},\,1,\,\tau_{e_i})+\sum\limits_{k\in K}\delta(k)\cdot\Big(1-\tau_d(\overline{\xi},\,k)\Big)\cdot d(\overline{\xi},\,k,\,\tau_d)}.$$

Hence,

$$\frac{\partial\overline{\pi}(\overline{\xi},\,\tau)}{\partial\tau_{e_i}\big(\overline{\xi},\,1\big)} = a_i \cdot \overline{\pi}(\overline{\xi},\,\tau) \cdot \frac{e_i(\overline{\xi},\,1,\,\tau_{e_i})}{\sum\limits_{i\in I}\Big(1-\tau_{e_i}(\overline{\xi},\,1)\Big)\cdot e_i(\overline{\xi},\,1,\,\tau_{e_i})+\sum\limits_{k\in K}\delta(k)\cdot\Big(1-\tau_d(\overline{\xi},\,k)\Big)\cdot d(\overline{\xi},\,k,\,\tau_d)}.$$

Set

$$\Pi(\overline{\xi},\,i,\,\tau) = \frac{e_i(\overline{\xi},\,1,\,\tau_{e_i})}{\sum\limits_{i\in I}\Big(1-\tau_{e_i}(\overline{\xi},\,1)\Big)\cdot e_i(\overline{\xi},\,1,\,\tau_{e_i})+\sum\limits_{k\in K}\delta(k)\cdot\Big(1-\tau_d(\overline{\xi},\,k)\Big)\cdot d(\overline{\xi},\,k,\,\tau_d)}.$$

Then

$$\frac{\partial\overline{\pi}(\overline{\xi},\,\tau)}{\partial\tau_{e_i}\big(\overline{\xi},\,1\big)} = a_i \cdot \overline{\pi}(\overline{\xi},\,\tau) \cdot \Pi(\overline{\xi},\,i,\,\tau).$$

We also know that

$$\frac{\partial \overline{q}(\xi,\,k,\,\tau)}{\partial \tau_{e_i}\left(\overline{\xi},\,1\right)} = \underbrace{\sum_{\xi' \in ET^+(\xi)} \frac{\partial \overline{\pi}(\xi',\ \tau)}{\partial \tau_{e_i}\left(\overline{\xi},\ 1\right)} \cdot \left(1 - \tau_d\left(\xi',\ k\right)\right) \cdot d\left(\xi',\ k,\ \tau_d\right) +}_{Changes\ in\ the\ Stochastic\ Discount\ \overline{\pi}(\xi',\,\tau)\ Factor\ for\ \xi'}$$

$$+ \underbrace{\sum_{\xi' \in ET^+(\xi)} \overline{\pi}(\xi',\ \tau) \cdot \left[\frac{\partial d\left(\xi',\ \tau_d\right)}{\partial \tau_{e_i}\left(\overline{\xi}\right)} \cdot \left(1 - \tau_d\left(\xi'\right)\right) - d\left(\xi',\ \tau_d\right) \cdot \frac{\partial \tau_d\left(\xi',\ \tau_d\right)}{\partial \tau_{e_i}\left(\overline{\xi}\right)}\right]}_{Changes\ in\ After-tax\ Dividends\ (1-\tau_d(\xi'))\cdot d(\xi',\,\tau_d)} \ \forall \overline{\xi} \in ET^+(\xi).$$

Substituting expression for $\frac{\partial \overline{\pi}(\overline{\xi},\,\tau)}{\partial \tau_{e_i}\left(\overline{\xi},\,1\right)}$ into the previous equation and taking into consideration that by Assumptions E4 and E5

$$\frac{\partial \tau_d}{\partial \tau_{e_i}} = \frac{\partial d}{\partial \tau_{e_i}} = 0,$$

we get

$$\boxed{\frac{\partial \overline{q}(\xi,\,k,\,\tau)}{\partial \tau_{e_i}\left(\overline{\xi},\,1\right)} = a_i \cdot \overline{\pi}(\overline{\xi},\,\tau) \cdot \Pi(\overline{\xi},\,i,\,\tau) \cdot \left(1 - \tau_d\left(\overline{\xi},\ k\right)\right) \cdot d\left(\overline{\xi},\ \tau_d\right) > 0\ \forall \overline{\xi} \in ET^+(\xi).\ \blacksquare}$$

References

Brown, J., N. Liang, and S. Weisbenner. Executive Financial Incentives and Payout Policy: Firm Responses to the 2003 Dividend Tax Cut. *Journal of Finance*, August 2007, 62 (4), 1935-1965.

DeLong, J.B. and K. Magin. The U.S. Equity Return Premium: Past, Present, and Future. *Journal of Economic Perspectives*, 2009, 23:1, 193-208.

Chetty, R., and E. Saez. Dividend Taxes and Corporate Behavior: Evidence from the 2003 Dividend Tax Cut. *Quarterly Journal of Economics*, August 2005, 120(3), 791-833.

Duffie, D. and W. Shafer. Equilibrium in Incomplete Markets: II. Generic Existence in Stochastic Economies. *Journal of Mathematical Economics* 1986, Volume 15: 199-216.

Edelstein, R. and K. Magin. Using the CCAPM with Stochastic Taxation and Money Supply to Examine US REITs Pricing Bubbles. T*he Journal of Real Estate Research*, 2016, Forthcoming.

Edelstein, R. and K. Magin. The Equity Risk Premium for Securitized Real Estate –The Case for U.S. Real Estate Investment Trusts. *The Journal of Real Estate Research*, 2013, Vol. 35:4.

Friedman, M., 1957. *A Theory of the Consumption Function.* Princeton University Press.

Lucas, R. E. Asset Prices in an Exchange Economy. *Econometrica*, 1978, 46:6, 1429-1445.

Magill, M. and Quinzii, M., 1996. *Theory of Incomplete Markets.* Volume I. MIT Press.

Magill, M. and W. Shafer, 1991. Incomplete markets, in *Handbook of Mathematical Economics*, *Volume IV*, Chapter 30, Pages 1523-1614, edited by W. Hildenbrand & H. Sonnenschein, Elsevier.

Magill, M. and W. Shafer. Characterization of Generically Complete Real Asset Structures. *Journal of Mathematical Economics* 1990, Volume 19: 167-194.

Magin, K. Properties of the Excess Demand Function in Finite Horizon Finance Economies with Stochastic Taxation, The Center for Risk Management Research Working Paper 2017.

(b)Magin, K. Generic Existence of Equilibria in Finite Horizon Finance Economies with Stochastic Taxation, The Center for Risk Management Research Working Paper 2015-10. http://riskcenter.berkeley.edu/working-papers/documents/2015-10_KonstantinMagin.pdf

(a)Magin, K., Equity Risk Premium and Insecure Property Rights, *Economic Theory Bulletin*, October 2015, Vol. 3, No. 2, pp 213-222, DOI 10.1007/s40505-014-0043-7.

Mas-Colell, A., M. D. Whinston and J. R. Green, 1995. *Microeconomic Theory.* Oxford University Press.

Quah, J. K.-H. Market Demand and Comparative Statics When Goods Are Normal. *Journal of Mathematical Economics,* 2003, 39, 317–333.

Sialm, C. Tax Changes and Asset Pricing. *American Economic Review*, 2009, 99:4, 1356-1383.

Sialm, C. Stochastic Taxation and Asset Pricing in Dynamic General Equilibrium. *Journal of Economic Dynamics and Control*, 2006, 30, 511-540.