Stochastic Taxation and REITs Pricing Bubbles: A Statistical Analysis

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Abstract

Using a modified Consumption Capital Asset Pricing Model (CCAPM) with stochastic taxation, we create estimates of fundamental values and fundamental overall rates of returns for United States Real Estate Investment Trusts (REITs) for our data sample, 1972 – 2013. Comparing actual, observed REITs prices (and overall rates of return) with model-generated fundamental values (and fundamental overall rates of return), we examine the presence of bubbles. For our purposes, for publicly traded equity REITs, we define a bubble to be the difference between actual stock market price (overall rates of return) and fundamental value (fundamental overall rate of return). United States REITs have, among other features, special rules governing dividend distributions and corporate taxation treatment that makes them an especially attractive and a preferred vehicle to test the presence of pricing and rate of return bubbles. Using this notion for bubbles, our study suggests that during the sample time horizon, United States REITs experienced statistically significant price and rates of return bubbles for a preponderance of the time.

Keywords: Bubbles, Equity Premium, REITs, Risk Aversion, Stochastic Tax, $t$-Statistics
I. Introduction

Economists and others have toiled and lucubrated for literally hundreds of years in order to identify, analyze, and explain asset market bubbles, booms and busts. From these efforts have emerged numerous studies and substantial and substantive academic and practitioner debates. This paper uses the Capital Consumption Asset Pricing Model (CCAPM) with stochastic taxation and the relatively idiosyncratic structural features of the US Equity REIT market to identify and statistically analyze bubbles for equity REITs between 1972 and 2013. As employed in this study, an economic bubble occurs when significant trading occurs at prices that appear to be inconsistent with intrinsic fundamental value.

Our analysis indicates that REITs price and overall rate of return bubbles are omnipresent and statistically significant for the preponderance of observations during our NAREIT sample time horizon, 1972-2013. Fundamental, intrinsic REIT value is derived from our prior analysis related to the CCAPM model with stochastic taxation, and assumed reasonable parametric modeling values. Our statistical results identify price and overall rate of return bubbles, and discuss plausible explanations for the observed bubbles.

How and why is this pricing-bubble study different from the multitude of predecessors. First, while it should almost go without saying, corporate managing, organizing, and planning as well as shareholder-investor decision-making tend to be tax sensitive. Any analysis of stock prices that does not take into account the impacts and effects of corporate and investor taxation is likely to be ignoring an important explanatory element for market behavior. Our analysis in this paper attempts to take into account that taxation is both stochastic and important; our analysis integrates stochastic taxation into an asset-pricing model employing the CCAPM theoretical framework.

Second, publicly traded Equity REITs vis-à-vis publicly traded C–corporations provide a natural laboratory for analyzing and evaluating bubbles for the following reasons: a) REITs, if they follow regulatory requirements, effectively do not pay taxation on net income at the corporate level; b) REITs are required to pay at least 90% of net income in the form of dividends. In essence, REITs distribute a substantial amount of cash flow in the form of dividends, and do not pay dividends from after-tax earnings, unlike “normal” profitable corporations. While corporations and shareholders – investors are typically carefully planning and monitoring taxation, in the case of REITs, corporate taxes de facto are inconsequential. This by itself simplifies our analytic tasks, permitting us to abstract from corporate taxation and employ analyses that take into account shareholder – investor taxation only.

The remainder of the paper is organized as follows. Section II provides a selective, targeted review of the voluminous bubbles, busts and booms literature as well as the stochastic taxation and the Equity Premium Puzzle, and how they relates to this paper. Section III is our theoretical section which defines and reviews the analytics for the CCAPM with stochastic taxation. In this section, we derive the theoretical fundamental pricing and overall rate of return equations. By employing reasonable parametric values for key variables, Section IV quan-
tify our theoretical model. Section V uses the CCAPM with stochastic taxation to compute theoretical-fundamental prices for REITs, which are then utilized to identify and statistically analyze possible asset-pricing bubbles. Section VI conducts a similar set of statistical analyses for bubbles for REITs overall rates of return. Section VII concludes.

II. A Targeted Selected Literature Review

This research paper spans, interfaces, and extends several well-developed, extensive and expansive financial economic research subject areas. Our analysis has benefited from an existing diverse, substantial set of research works on economic booms, busts and bubbles. Our paper is especially influenced by several interesting and important analyses pertaining to real estate market booms and busts. In addition, our research is intertwined with asset pricing models with concomitant issues, such as the Equity Premium Puzzle, the coefficient of relative risk aversion for investors, and the impacts of stochastic taxation on investment decision-making. We will now provide a very brief selective review of pertinent prior research.

Booms, busts, and bubbles

As a starting point for understanding bubbles, one should acknowledge the monumental contribution made by Charles Kindleberger in his book, *Manias, Panics and Crashes*. Kindleberger, an extraordinary economic historian, traces and analyzes various bubbles episodes across history in which economic outcomes are speculative, and in no way reflective of the underlying fundamental economic values. His analyses of the northern European tulipmania in 1636-1637, and the English South Seas bubble in the early 1700s are captivating, and reminiscent of alleged bubbles through history to the modern day. During the peak of Tulipmania in March 1637, tulip bulbs were transacting for values that were about 10 times the annual income of a skilled craftsman. Suddenly, in 1637 the tulip bulb values started to decline precipitously to levels of 2 to 5% of their peak. The South Sea Company, a British stock company founded in 1711, was a public-private partnership that was granted a monopoly for British trade with South America. However, England was at war with Spain, and Spain controlled South America. Hence, there was little real prospects that trade would take place for the company in South America. However, the company stock skyrocketed as it expanded its transactions in British government debt. The company’s value peaked in 1720, before collapsing to approximately the original flotation price; hence, the so-called South Sea Bubble. Because of the public outcry, Britain in 1720 past the Bubble Act which forbade the creation of such stock companies without royal chartering.

Reinhart and Rogoff (2009), in their book *This Time Is Different: Eight Centuries of Financial Folly* explore the interrelationship between speculative bubbles, inflation, debt, and monetary crises over the last 800 years. As in the Kindleberger book, they examine several historical episodes of speculative bubbles, and attribute many the issues to unrealistic expectations, speculative
behavior, and over leverage. They conclude that public and private sector mis-handling of debt is frequently the cause for these speculative bubbles. The most recent work by Mian and Sufi (2014) examines how the Great Recession of 2008 and the housing market were intertwined with the financial sector, especially because of overzealous mortgage debt issuance.

Robert Shiller (2005) in the second edition of his book, *Irrational Exuberance*, a phrase coined by a now infamous quote from then Federal Reserve chief Alan Greenspan, develops several arguments demonstrating how the stock market was “over valued.” In this book, he also suggests that the U.S. real estate market at the time (2005) was likely to be a bubble, a bubble that was punctured three or four years after the book!

It is often thought that "bubbles" are relatively short-term phenomena, and quickly come down to earth. The U.S. residential real estate bubble commenced in 2001, and did not reach its peak until 2006. This real estate bubble was replicated in many parts of the world, coming to a crashing halt in 2007 (see Bardhan, Edelstein, and Kroll (2012)). In fact, bubbles can last decades as documented by Ambrose, et al (2012). In their study of 300+ years of housing price behavior in Holland, they find that bubbles can have elongated lives, lasting 70 or 80 years where housing prices systematically do not reflect rental fundamentals.

While there are many explanations, ranging from overleverage — loose monetary policy, crowd herding, animal spirits, and heterogeneous expectations, the upshot is that observed asset prices can differ significantly, sometimes for prolonged periods, from intrinsic underlying fundamental economic value. This said, it is sometimes difficult to determine what is the underlying intrinsic fundamental economic value, irrespective of the mechanism causing the speculative bubble.

**Asset Prices, Stochastic Taxation and the Equity Premium Puzzle**

Paradoxically, almost no research has been done about the effects of stochastic taxes on asset prices and allocations. The research that has been done was primarily motivated by the Equity Premium Puzzle. The Equity Premium Puzzle was originally identified by Mehra and Prescott (1985), using historical data for the stock market portfolio $\beta = 1$. The traditional Capital Consumption Asset Pricing Model (CCAPM), with an isoelastic Constant Relative Risk Aversion (CRRA) utility function and an expected equity risk premium of 6% for the S&P 500, using average historical stock returns, produces a coefficient of relative risk aversion of roughly 47.6. This unbelievably high coefficient of relative risk aversion constitutes the so-called "Equity Premium Puzzle". There have been many attempts to resolve the Equity Premium Puzzle. The introduction of taxation into the standard macroeconomic models seemed to pave one of the most promising ways to approach the puzzle. McGrattan and Prescott (2005), Sialm (2006) and (2011) were among the first to introduce taxation into the

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4See DeLong and Magin (2009) for a review, for example.
General Equilibrium models. However, their work does not resolve or directly address the puzzle.

Magin (2014), recognizing that taxation uncertainty plays a major role for investors, introduced a modified CCAPM with a stochastic tax rate $\tau$ imposed on the income and capital wealth of stock holders. Using this modified model, he finds that for an average investor, who realizes after-tax dividend income as well as short-term and long-term gains in accordance with historical patterns, the coefficient of relative risk aversion is 3.76. Since earlier studies imply that a coefficient of relative risk aversion, $\alpha$, between 2 and 4 would seem reasonable, the Magin estimate for $\alpha = 3.76$ is believable.

The risk premium puzzle for asset classes other than $\beta = 1$ stock market portfolios has been largely unexplored. The known exceptions for real estate assets are Shilling (2003) and Edelstein and Magin (2013, 2014). In his study, Shilling (2003) deploys the CCAPM and two different real estate value data sets; but he does not take into account the possible impacts of taxation. He confirms the existence of the equity risk premium puzzle for real estate assets, and concludes that the "puzzle" is even more pronounced for real estate than for general stock market.

On the other hand, using a novel modeling twist by applying the CCAPM with stochastic taxation to NAREIT data, Edelstein and Magin (2013) demonstrate that, for a range of reasonable stochastic tax burdens, the coefficient of relative risk aversion for US Equity REITs shareholders is likely to fall within the interval of 4.32 to 6.29, values significantly lower than those reported in most prior studies for real estate and other asset markets.

III. Using the CCAPM for Determining Fundamental REITs Values and Rates of Return

To resolve the Equity Premium Puzzle for securitized real estate and to analyze REITs pricing, Edelstein and Magin (2013, 2014) use the representative agent CCAPM with stochastic taxation derived by Magin (2014). They consider the investor’s optimization problem:

$$\max_{\{c_{t+T}\}_{T=0}^{\infty}} E \left[ \sum_{T=0}^{\infty} \beta^T u(c_{t+T}) \right],$$

where $0 < \beta < 1$ and $u(\cdot)$ is such that $u'(\cdot) > 0$ and $u''(\cdot) < 0$, subject to

$$c_{t+T} = \sum_{k=1}^{n} (p_{kt+T} + (1 - \tau_{kt+T}^d) d_{kt+T} z_{kt+T} - \sum_{k=1}^{n} p_{kt+T} z_{kt+T+1},$$

where $\tau_{kt+T}^d$ is the effective stochastic tax rate imposed on dividends of asset $k$ at period $t$.

\textsuperscript{5}Mehra (2003), Mehra and Prescott (2003)
Then in equilibrium, assuming the CRRA utility function

\[ u(c) = \frac{c^{1-a}}{1-a}, \]

where \( a \) is the coefficient of constant relative risk aversion and the Transversality Condition

\[ \lim_{T \to \infty} E \left[ b^T \left( \frac{(1-\tau^d_{k+T})d_{kt+T}}{(1-\tau^d_t)d_{kt}} \right)^{1-a} p_{kt+T} \right] = 0 \quad \forall k = 1, \ldots, n \]

implies that the price of an asset \( k \) at period \( t \) is given by

\[ p_{kt} = E \left[ \sum_{T=1}^{\infty} b^T \left( \frac{(1-\tau^d_{k+t+T})d_{kt+T}}{(1-\tau^d_t)d_{kt}} \right)^{1-a} \right] \cdot (1-\tau^d_t)d_{kt} \quad \forall k = 1, \ldots, n. \quad (3) \]

Assuming further that

\[
\ln(b(\frac{(1-\tau^d_{t+i+1})d_{kt+i+1}}{(1-\tau^d_{t+i})d_{kt+i}})^{1-a}) = \ln(b(\frac{\mu_{c,i}}{\sigma_{c,i}})^{1-a}) \sim N(\mu_c, \sigma_c) \quad \forall i = 0, \ldots, \infty,
\]

where

\[
\mu_c = E \left[ \ln(b(\frac{\mu_{c,i}}{\sigma_{c,i}})^{1-a}) \right] = \ln(b) + (1-a) \cdot E \left[ \ln(\frac{\mu_{c,i}}{\sigma_{c,i}}) \right],
\]

\[
\sigma_c^2 = VAR \left[ \ln(b(\frac{\mu_{c,i}}{\sigma_{c,i}})^{1-a}) \right] = (1-a)^2 \cdot VAR \left[ \ln(\frac{\mu_{c,i}}{\sigma_{c,i}}) \right],
\]

we obtain that the theoretical (implied by the tax-adjusted CCAPM) price \( p_{kt} \) per share of an equity REIT \( k \) at period \( t \) is given by

\[ p_{kt} = \left[ e^{\mu_c + \frac{1}{2} \sigma_c^2} \right] \cdot (1-\tau^d_{re,t}) d_{kt}, \quad (4) \]

where \( \tau^d_{re,t} \) be the effective stochastic tax rate imposed on the dividends paid to Equity REITs holders.

Conventionally, we define the asset-pricing bubble as the difference between the actual market price of an asset and its theoretical price. Therefore, the asset-pricing bubble for an arbitrary asset \( k \) at period \( t \) is given by

\[ B_{kt} = \overline{p}_{kt} - p_{kt} \quad (5) \]
where $p_{kt}$ is the actual market price of an arbitrary asset $k$ at period $t$.

Let us derive now the theoretical rate of return $R_{kt+1}$ implied by our tax-adjusted CCAPM. Equation (4) from above implies that as long as the stochastic parameters $\mu$ and $\sigma$ are constant, the price/after-tax dividend ratio $(1-\tau_{re,t})d_{kt}$ remains unchanged. Set

$$\lambda = \frac{p_{kt}}{(1-\tau_{re,t})d_{kt}}.$$ 

Therefore,

$$R_{kt+1} = \frac{p_{kt+1} + (1-\tau_{re,t+1})d_{kt+1}}{p_{kt}} = \lambda \frac{(1-\tau_{re,t+1})d_{kt+1} + (1-\tau_{re,t+1})d_{kt+1}}{(1-\tau_{re,t})d_{kt}} = \frac{\lambda + 1}{\lambda} \frac{d_{kt+1}}{(1-\tau_{re,t})d_{kt}}.$$ 

Thus, the theoretical overall rate of return will be

$$R_{kt+1} = \frac{\lambda + 1}{\lambda} \frac{(1-\tau_{re,t+1})d_{kt+1}}{(1-\tau_{re,t})d_{kt}}. \quad (6)$$ 

Similarly, we define the rate of return bubble as the difference between the actual rate of return on an asset and its theoretical rate of return. Therefore, the rate of return bubble for an arbitrary asset $k$ at period $t$ is given by

$$BR_{kt} = R_{kt} - \overline{R}_{kt} \quad (7)$$

where $\overline{R}_{kt}$ is the actual rate of return on an arbitrary asset $k$ at period $t$.

**IV. Quantifying Theoretical Model**

In order to generate a series of theoretical prices (equation (4)) and theoretical overall rates of return (equation (6)), we need to assume reasonable values for the key parameters. Historically,$^6$

$$E\left[\ln\left(\frac{c_{t+1}}{c_t}\right)\right] = 0.02,$$

$$VAR\left[\ln\left(\frac{c_{t+1}}{c_t}\right)\right] = 0.00125.$$ 

Set

$$b = \frac{1}{\tau_{f}} = \frac{1}{1.01} = 0.99.$$ 

Therefore, we estimate

\[ \mu_c = \ln(0.99) + (1 - a) \cdot 0.02, \]
\[ \sigma_c^2 = (1 - a)^2 \cdot 0.00125. \]

Thus,
\[ \mu_c + \frac{1}{2} \sigma_c^2 = \ln(0.99) + (1 - a) \cdot 0.02 + \frac{1}{2} \cdot (1 - a)^2 \cdot 0.00125. \]

Hence,
\[ e^{\mu_c + \frac{1}{2} \sigma_c^2} = e^{\ln(0.99) + (1 - a) \cdot 0.02 + \frac{1}{2} \cdot (1 - a)^2 \cdot 0.00125}. \]

Edelstein and Magin (2013) estimated that for Equity REITs holders

\[ 4.32 < a < 6.29. \]

Therefore, it is reasonable for the purposes of our analyses to set

\[ a = 5. \]

Using equation (4), we obtain the following expression for the price of Equity REITs

\[ p_{kt} = \left[ \frac{e^{\ln(0.99) + (1 - 5) \cdot 0.02 + \frac{1}{2} \cdot (1 - 5)^2 \cdot 0.00125}}{1 - e^{\ln(0.99) + (1 - 5) \cdot 0.02 + \frac{1}{2} \cdot (1 - 5)^2 \cdot 0.00125}} \right] \cdot (1 - \tau^{d}_{re \cdot t}) \cdot d_{kt} \quad (8) \]

As in Edelstein and Magin (2013, 2014), we are assuming that the typical investor in REITs, who has below average ordinary income tax rates, pays an overall effective dividend tax rate \( \tau^{d}_{re \cdot t} \) of half of that of an investor in general stocks.\(^7\)

Similarly, quantifying equation (6), we obtain

\[ R_{kt+1} = \frac{\left[ \frac{e^{\ln(0.99) + (1 - 5) \cdot 0.02 + \frac{1}{2} \cdot (1 - 5)^2 \cdot 0.00125}}{1 - e^{\ln(0.99) + (1 - 5) \cdot 0.02 + \frac{1}{2} \cdot (1 - 5)^2 \cdot 0.00125}} \right] + 1}{\left[ \frac{e^{\ln(0.99) + (1 - 5) \cdot 0.02 + \frac{1}{2} \cdot (1 - 5)^2 \cdot 0.00125}}{1 - e^{\ln(0.99) + (1 - 5) \cdot 0.02 + \frac{1}{2} \cdot (1 - 5)^2 \cdot 0.00125}} \right]} \cdot \frac{(1 - \tau^{d}_{re \cdot t+1})}{(1 - \tau^{d}_{re \cdot t})} \cdot d_{kt+1}, \quad (9) \]

V. Statistical Analysis of REITs Prices

Let us turn now to the statistical analysis of REITs Prices. Figure 1 below charts theoretical (fundamentals generated by equation (8)) and actual market prices for Equity REITs for the period of 1972–2013\(^8\). A cursory examination of the arrayed data suggests that fundamental values, as determined by the model, deviate substantially and frequently from the actual observed market prices.

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\(^7\) Effective dividend tax rates for investors in general stocks can be found at http://users.nber.org/~taxsim/marginal-tax-rates/af.html

\(^8\) Calculations are based on monthly NAREIT ALL EQUITY REITs INDEX data for Equity REITs prices and dividends.
As discussed above, this difference between fundamental value and actual prices constitutes our definition of a “bubble.” Applying the standard $t$-statistical tests for these data, we discover that the deviations between fundamental values and observed prices are statistically significantly different at the 5% level for 21 of the 42 years.\footnote{Let $\bar{p}_{kt} \in \mathbb{R}^n$ and $p_{kt} \in \mathbb{R}^n$ are the actual and theoretical prices of asset $k$ at period $t$ respectively. Then we define the $t$-statistics as follows}

\[
\left\{ \frac{\bar{p}_{kt} - p_{kt}}{\sqrt{\frac{\text{var}(\bar{p}_{kt})}}}, \right\}_{t=1972}^{2013}
\]

where

\[
\bar{p}_{kt} = \sqrt{\text{var}(\bar{p}_{kt})}.
\]
undervalued in the marketplace. The two other years when there were statistically significant negative deviations between actual prices and fundamental values occurred in 1974 and 1990. These latter findings upon reflection are quite understandable. In 1974, REITs were out of favor in the marketplace, and REIT prices were in collapse. In contrast, in 1990 the so-called “Dot-Com” boom was at its pinnacle, and cash flowing real estate investments, such as REITs, were not in great demand.

VI. Statistical Analysis of REITs Rates of Return

Asset prices, such as publicly traded equity stock, are typically believed to be statistically nonstationary. This may create bias and inconsistency for our simple asset pricing comparison statistical tests for bubbles. In contrast overall rates of return are generally considered to be stationary. Figure 2 below charts theoretical (fundamentals generated by equation (9)) and actual rates of return for Equity REITs for the period of 1973–2013\(^\text{10}\). Therefore, for our 1973-2013 time series sample, we create two vectors of overall rates of return using the actual NAREIT index data as well as our theoretical pricing model, equation (9). The end product from this exercise is 2 vectors with 41 time series observations for the overall rates of return for years, 1973-2013.

Employing a similar \(t\)-statistical test previously utilized for REIT pricing, we compare the differences between the actual, observed REIT overall rate of returns and the theoretical model generated overall rates of return. The statistical analysis for the rates of return comparisons engenders similar findings vis-à-vis the pricing comparisons. The upshot of this statistical analysis is that for 35 of the 41 years actual market overall rates of return were statistically, at the 5% level or better, different from the overall rates of return generated by our theoretical benchmark model. There are 16 statistically significant positive bubbles; and there are 19 statistically significant negative bubbles. As with the pricing statistical tests, there are prominent negative rate of return bubbles in 1973 – 74, a bleak era for real estate, in general, and REITs in particular; in 1980-81, there is another negative bubble, during a sub-period of general economic malaise and extraordinarily high interest rates; a third two-year period for statistically significant negative bubbles occurred in 1989-90; during 2001-2002, a period of economic turmoil related to 9/11, REITs experienced another two year stint characterized by negative rate of return bubbles; and finally between 2005-2007, a precursor and beginning of the great financial and economic recession, there was a significant negative bubble for rates of return. During recent history, 2000-2013, for 12 of the 14 years, REIT actual overall rates of return were statistically significantly different from the theoretical model generated rate of return; and evenly divided with six negative and six positive bubbles.

\(^{10}\)Calculations are based on monthly NAREIT ALL EQUITY REITs INDEX data for Equity REITs prices and dividends.
Figure 1:
VII. Conclusion

This paper identifies United States Real Estate Investment Trusts (REITs) price and rate of return bubbles using the NAREIT data base, 1972-2013. For this analysis, a bubble is defined for publicly traded REITs to be at a point in time the difference between the actual, observed stock price (overall rate of return) and the intrinsic, fundamental value (fundamental overall rate of return). Fundamental, intrinsic value is derived by utilizing our earlier research employing the modified Consumption Capital Asset Pricing Model (CCAPM), including stochastic taxation, with reasonable parametric assumptions. For analytical purposes, REITs provide a preferred natural laboratory experiment for bubble testing because of the rules governing net income taxation and dividend distributions; in essence, REITs basically are pass through vehicles without taxation at the entity level, permitting our theoretical modeling to focus upon the inclusion of shareholder – investor taxation, without the additional complications of investment vehicle taxation. Taken together, our modified CCAPM and REITs create a set of cleaner statistical analyses for testing for the presence of price and overall rate of return bubbles.

Our analysis suggests that REITs price bubbles and overall rate of return bubbles are omnipresent and statistically significant during our sample time horizon. While we provide plausible macroeconomic rationales for the various sequences of bubbles, our research should be characterized as identifying, but not necessarily explaining the root causes of these bubbles. We leave the determining of causal explanations and relationships between REIT bubbles and other variables as a task for future research.
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