Fragility of risk measures and other computed metrics

Many metrics of interest in risk management, like the value-at-risk or conditional value-at-risk of a position, or the price of an option, are computed using a stochastic model which has been calibrated using historical data. The reliability of these calculations depends strongly on the underlying model and the calibrated parameters.

The objective of this proposal is to develop methods for evaluating the fragility of these calculations to perturbations in the model and the data. For example, we would like to be able to answer questions like:

- How sensitive is the conditional value-at-risk (CVaR) calculation for the current portfolio to the assumptions of normality? Is it the case that a slight perturbation of the distribution (which makes the tails heavier, say) will drastically change the CVaR calculation (so the model is “fragile”), or is it the case that the CVaR value will remain within “tolerable bounds” for a fairly large perturbation of the model?
- How sensitive are your computed option prices to the same perturbations of the model discussed above?
- How sensitive are computations based on choices of marginal distributions and a copula to perturbations in these models?
- Is it possible to identify the sort of perturbations to which the price/risk measure calculations are most sensitive?

We believe that a systematic method for evaluating model and metric fragility will be useful for identifying vulnerabilities in a model. (How “safe” is a portfolio position if a small fattening of the tails will cause a dramatic increase in its CVaR? Likewise, is an option “correctly priced” if a small change in the model will dramatically change the price). In particular, it could be used to inform the user that his/her calculations are getting fragile, and the way in which the fragility is arising.
Detailed description of proposed research

Our research will center about computing the fragility of derivative price and risk-measure computations. More specifically, we are concerned with fragility from two sources:

- **Fragility within a parametric family:**
  - Suppose that a calculation (of a price or risk-measure) is based on assumptions of normality, and that the parameters of the model (covariance matrix and means) have been calculated on the basis of parameter estimates. How sensitive is the metric to changes in the basis of the parameters? *Which deviations is it the most sensitive to?*

- **Non-parametric deviations:**
  - How fragile is the calculation with respect to the assumption of normality in the distribution? Do metrics change drastically if the family of distributions changes? (e.g. if tails get heavier). *What is the sort of perturbation is the metric most sensitive to?*
  - A dynamic model might assume a log-normal (or some other class) of distributions. How sensitive is the calculation to the assumption of this parametric family? Again, *what sort of deviations will the computations be most sensitive to?*
  - How sensitive is the price of a credit derivative to the specification of the copula and the marginals.

In all cases, the goal is to develop efficient methods for computing charts which relate “allowable change in the computed number/metric” (on the vertical axis) to the “size of the worst case model deviation” which achieves each of the changes (on the horizontal). Such charts can alert risk-managers that a small worst-case deviation can causes a drastic change in the metric.

Understanding worst-case sensitivities to specification of mean and covariance matrix (but still assuming that the distribution is normal) will be carried out using tools from convex optimization, an area that Laurent El Ghaoui is an expert. This work builds on his expertise in calculating *worst-case-VaR* within the family of normal distributions. Calculation of sensitivities to perturbations in the actual distribution will be based on deviation measures such as relative entropy, and builds on the work of Andrew Lim and George Shanthikumar on robust decision making where these deviation measures are used.