1. Project Summary

The Troubled Assets Relief Program (TARP) authorized and appropriated by Congress in 2008 enables the Secretary of the Treasury to purchase on the government’s behalf so-called “troubled assets” the purchase of which is necessary in order to promote economic stability. Troubled assets include mortgages, pools of mortgages, mortgage-backed securities, and further derivative securities.

The most difficult problem of implementation faced by the TARP is that of the pricing of “troubled” assets. Liquid markets for most of them simply do not exist. It is of vital importance to the government and the country to determine how to establish the appropriate values at which these securities should be purchased.
The main goal of this proposal is to develop a methodology to price “troubled” assets. We are going to start with pricing individual “troubled” mortgages. Then we shall proceed to pricing pools of mortgages. Finally, we should be able to price collateralized debt obligations.
2. Project Description

Coleman Fung Center Grant Proposal: The Pricing of “Troubled” Assets

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Konstantin Magin, Assistant Researcher

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April 30, 2009

The Problem

The Troubled Assets Relief Program (TARP) authorized and appropriated by Congress in 2008 enables the Secretary of the Treasury to purchase on the government’s behalf so-called “troubled assets” created on or before March 14, 2008 the purchase of which is necessary in order to promote economic stability. Troubled assets include mortgages, pools of mortgages, mortgage-backed securities, and further derivative securities.

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Fortunately for this project, new investment vehicles are currently being
created. Standard & Poors licensed MacroMarkets to create products based
on the Case-Shiller Home Price Indexes. The CSI-based ETF will begin
trading on the NYSE Arca this Fall.1 According to ETF Express, the Case-
Shiller Home Price Index is aimed at tracking the price path of single family
homes using the repeat sales technique developed by Karl Case and Robert
Shiller. The index is weighted in 10 metropolitan areas throughout the U.S.
and includes Boston, Chicago, Denver, Las Vegas, Los Angeles, Miami, New

1http://online.wsj.com/article/SB123897667301591301.html
York (commuter index), San Diego, San Francisco and Washington, D.C.²

In addition, in 2006 the Chicago Mercantile Exchange (CME) launched futures and options written on S&P/Case-Shiller Home Price Index (CSI). The modelling and pricing of these new vehicles was pioneered by Case, Shiller and Weiss (1993), (1998), Shiller and Weiss (1999) ³

The main goal of this proposal is to develop a methodology to price “troubled” assets. The first step will be to apply the standard Black-Scholes methodology to the problem of valuing “troubled” assets, as outlined in the model section below. What prices for “troubled” assets would be consistent with futures prices for the CSI as traded on markets? Using market prices of futures contacts written on S&P/Case-Shiller Home Price Index traded on

²http://www.etfexpress.com/articles/pdf_page.jsp?content_id=267240
CME we can obtain the implied market volatility to estimate the variance of this index. Then we are going to use this estimate to calculate the Black-Scholes price of call options written on CSI-based ETF. Then we can design a hedging strategy that replicates mortgages going into default next period. This strategy is going long on this mortgage’s collateral (shares of CSI-based ETF) and short on the call option written on CSI-based ETF. So, we obtain prices of mortgages going into default. This, however, can only be a first and preliminary run at the problem. So, we are going to generalize the model. We are going to model the situation, where there is an uncertainty whether or not mortgage is going into default. We shall model the number of mortgages going into default next period as a function of future values of CSI-based ETF and other macroeconomic variables. Constructing this model will help us to capture the very essence of the current mortgage crisis and open the door to pricing pools of mortgages and collateralized debt obligations.

This methodology will be developed collaboratively by J. Bradford De-Long, Professor of Economics (the PI), and Konstantin Magin, Lecturer in Economics and Lecturer in Business at U.C. Berkeley. We anticipate that
at least two of the papers to emerge from the project will become single-authored publications by Konstantin Magin. The project will run for one calendar year, from the summer semester of the 2009 through the fall and spring semester of the 2009-10 academic year.
The Model

Definition: We call loan a troubled asset if borrower is not making minimum payment.

The following theorem below proposes hedging strategy to replicate and price troubled asset.

Theorem: Suppose investor maximizes

\[
U = E \left[ \frac{1}{1-\lambda} C_t^{1-\lambda} + b \frac{1}{1-\lambda} C_{t+1}^{1-\lambda} \right] .
\]

Let \( PL_t \) be the price of troubled asset at period \( t \), \( p_t \) be the price of the loan’s collateral at period \( t \). Suppose also that \( x = \ln \left( \frac{p_t+1}{p_t} \right) \) and \( y = \ln \left( b \left( \frac{c_t+1}{c_t} \right)^{1-\lambda} \right) \) are bivariate normally distributed with expectations

\[
(E[x], E[y]) = (\mu_x, \mu_y)
\]

and the variance-covariance matrix

\[
V = \begin{pmatrix}
\sigma_x & \rho \sigma_x \sigma_y \\
\rho \sigma_x \sigma_y & \sigma_y
\end{pmatrix}.
\]

Suppose also that \( PR_{t+1} \) is the loan’s principal balance left at period \( t+1 \).

Then
\[ PL_t = p_t \left( 1 - N(Z + \sigma) \right) + \frac{PR_{t+1}}{1+r_f} N(Z), \]

where

\[ N(z) = \int_{-\infty}^{z} \frac{1}{\sqrt{2\pi}} e^{-\frac{v^2}{2}} dv \]

and

\[ Z = \left( \ln \frac{p_t}{p_{R_{t+1}}} + \ln R_f \right) - \frac{1}{2} \sigma. \]

**Proof**: Note first that the lender’s period \( t + 1 \) pay off on troubled loan originated at period \( t \) is \( \min[PR_{t+1}, p_{t+1}] \), where \( PR_{t+1} \) is the loan’s principal balance left at period \( t+1 \) and \( p_{t+1} \) is the price of the loan’s collateral at period \( t + 1 \). We know that the price of this risky bond \( PL_t \) with return \( R = 1 + r = \min[PR_{t+1}, p_{t+1}] \) is given by

\[ PL_t = p_t - \text{Call}(p_t, PR_{t+1}). \]

We also know that

\[ \text{Call}(p_t, PR_{t+1}) = p_t N(Z + \sigma) - \frac{PR_{t+1}}{1+r_f} N(Z), \]

where
\[ Z = \ln \frac{p_t \ln R_{t+1}}{\sigma} + \ln R_f - \frac{1}{2} \sigma. \]

Hence,

\[ PL_t = p_t - p_t N(Z + \sigma) + \frac{PR_{t+1}}{1 + R_f} N(Z) = \]

\[ = p_t (1 - N(Z + \sigma)) + \frac{PR_{t+1}}{1 + R_f} N(Z). \]
3. Curricula Vitae

J. BRADFORD DeLONG

A. Education

Ph. D., Department of Economics, Harvard University, Cambridge, MA (June 1987).

B. A. summa cum laude, Committee on Degrees in Social Studies, Harvard University, Cambridge, MA (June 1982).

B. Positions

Chair, Political Economy of Industrial Societies Major, University of California at Berkeley, Berkeley, CA (July 2001-present).

Professor, Department of Economics, University of California at Berkeley, Berkeley, CA (July 1997-present).

Associate Professor, Department of Economics, University of California at Berkeley, Berkeley, CA (July 1993-June 1997).

Frederick S. Danziger Associate Professor, Department of Economics, Harvard University, Cambridge, MA (July 1991-June 1993).

Assistant Professor, Department of Economics, Harvard University, Cambridge, MA (July 1988-June 1991).
Assistant Professor, Department of Economics, Boston University, Boston, MA (July 1987-June 1988).


Research Associate, National Bureau of Economic Research, Cambridge, MA (October 1995-present).
C. Principal Academic Research Publications


J. Bradford DeLong (2003), ”Bequests: An Historical Perspective,” in


J. Bradford DeLong (2000), ”Overstrong Against Thyself: War, the State, and Growth in Europe on the Eve of the Industrial Revolution” , in Mancur Olson and Satu Kahkohen, eds., A Not-So-Dismal Science: Development


KONSTANTIN A. MAGIN

Education:


Employment:

2007-present, Lecturer, UC Berkeley

2003-2007, Post-doctoral Fellow, UC Berkeley

Publications:


Papers:


“Finance and the Future of Nanotechnology: Looking Forward by Looking Back”

“Understanding the 1987 Stock Market Crash: An Analysis”
4. Progress Report

No research has been funded by the center.
5. Budget

The budget requested is $55,577: $36,360 for salary for a 75% appointment of Konstantin Magin as an Assistant Researcher and $6,161 for fringe benefits; and $13,056 for a one-ninth one-month summer salary for J. Bradford DeLong.